

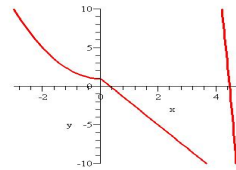
General Function Information:

1. Function: A *function* is a rule that associates an element of one set, called the *domain* uniquely with an element of another set called the *range*.
2. Inverse Functions: $f(x)$ and $g(x)$ are *inverse functions* if $f(g(x)) = x$ and $g(f(x)) = x$. We normally write $g(x) = f^{-1}(x)$. The graph of the inverse of a function is the reflection in the line $y = x$ of the graph of the function.
3. Even and Odd Functions:

- A function is *even* if $f(-x) = f(x)$, its graph is symmetric about the y -axis.
- A function is *odd* if $f(-x) = -f(x)$, its graph is symmetric about the origin.

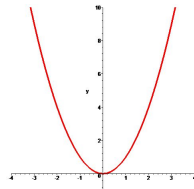
4. Piecewise Defined Functions: For Example

$$f(x) = \begin{cases} x^2 + 1 & x \leq 0 \\ -3x + 1 & 0 < x \leq 4 \\ -x^3 + 20x & x > 4 \end{cases}$$

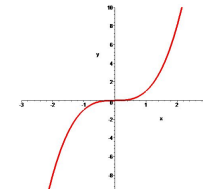


5. Algebraic Functions:

- Linear Function:
 - Slope: $m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$
 - Point Slope Form: $(y - y_0) = m(x - x_0)$ or $y = m(x - x_0) + y_0$
 - Slope Intercept Form: $y = mx + b$
- Power Function: $y = x^r$, where $r \in \mathbb{R}$
- Polynomial: $y = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$, where a_i is any number, $a_n \neq 0$, and all the exponents are non-negative integers. If $a_n = 1$ the polynomial is called *monic*.
 - The *degree* of a polynomial is the highest power of the variable.



Even Degree Polynomial

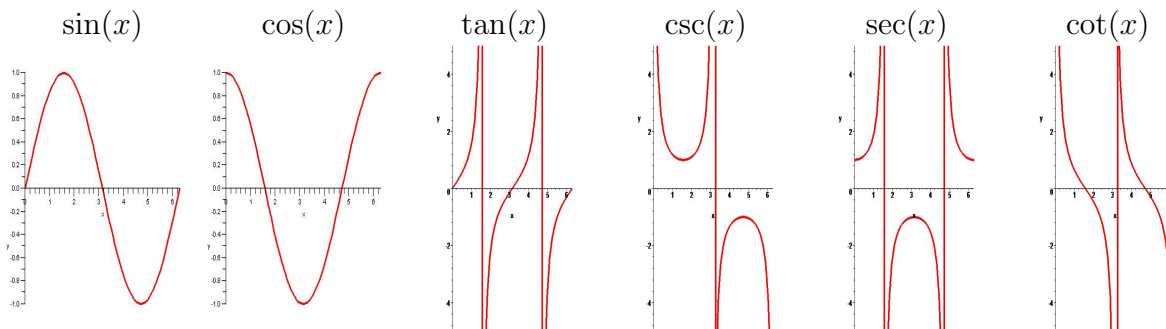


Odd Degree Polynomial

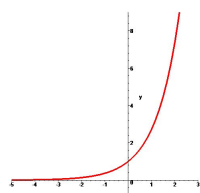
- A *root* or *x-intercept* of a polynomial are the places it crosses the x -axis which is the same as the values of x for which the polynomial equals 0.
 - The number of real roots of a polynomial is less than or equal to its degree.
 - The *y-intercept* of a polynomial is where it crosses the y -axis and is the constant term of the polynomial.
- Rational Functions: $y = \frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials.
 - General Algebraic Function: Any combination of x 's involving addition, subtraction, multiplication, division, and raising to fixed powers.

6. Transcendental Functions:

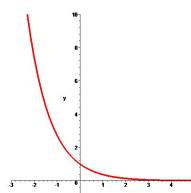
- Trigonometric Functions: $\sin(x)$, $\cos(x)$, $\tan(x)$, $\sec(x)$, $\csc(x)$, and $\cot(x)$.



- Exponential Functions: $y = a b^x$ where a and b are real numbers, and b , the base, is greater than 0.

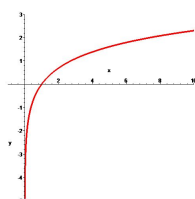


$$b^x, b > 1$$



$$b^x, 0 < b < 1$$

- Logarithmic Functions: $y = \log_b(x)$ is the inverse function of $y = b^x$, and b , the base, is a real number greater than 0.



$$\log_b(x), b > 1$$

- Common Log: $y = \log(x)$, the logarithm base 10.
- Natural Log: $y = \ln(x)$, the logarithm base e .

Quadratics, Polynomials, and Rational Functions:

- Quadratics: A *quadratic polynomial* is a polynomial of the form

$$y = ax^2 + bx + c \text{ or } y = a(x - r)(x - s), \quad a \neq 0$$

- Roots: A *root* or *x-intercept* of a polynomial are the places it crosses the *x-axis* which is the same as the values of *x* for which the polynomial equals 0.
 - For a quadratic written as $y = a(x - r)(x - s)$ the roots are $x = r$ and $x = s$.
 - If the quadratic is monic, $y = x^2 + bx + c$, then the product of the roots equals c and their sum is $-b$. That is if

$$x^2 + bx + c = (x - r)(x - s)$$

then $-b = (r + s)$ and $c = (r \cdot s)$.

- In general the roots of a quadratic can always be found using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

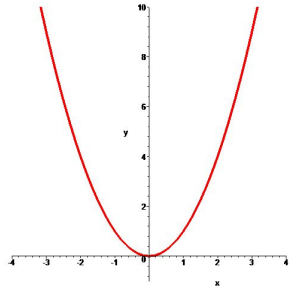
- Polynomial Long Division: Examples

$$\begin{array}{r} x^2 + 5x + 8 \\ x - 2 \overline{) x^3 + 3x^2 - 2x - 16} \\ \underline{x^3 - 2x^2} \\ 5x^2 - 2x - 16 \\ \underline{5x^2 - 10x} \\ 8x - 16 \\ \underline{8x - 16} \\ 0 \end{array}$$

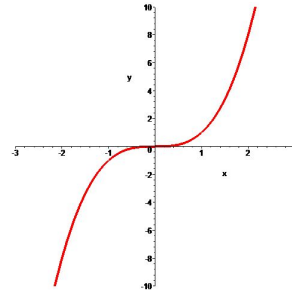
$$\begin{array}{r} x^3 - 9x^2 + 17x - 22 \\ x^2 + 2x + 1 \overline{) x^5 - 7x^4 + 3x^2 - 9} \\ \underline{x^5 + 2x^4 + x^3} \\ -9x^4 - x^3 + 3x^2 - 9 \\ \underline{-9x^4 - 18x^3 - 9x^2} \\ 17x^3 + 12x^2 - 9 \\ \underline{17x^3 + 34x^2 + 17x} \\ -22x^2 - 17x - 9 \\ \underline{-22x^2 - 44x - 22} \\ 27x + 13 \end{array}$$

- Factors and Factoring: The factors of a polynomial are the other polynomials that divide evenly into it. For example from above $y = x - 2$ is a factor of $y = x^3 + 3x^2 - 2x - 16$. If $x = r$ is a root of a polynomial then $y = x - r$ is a factor and in general can be factored out using polynomial long division.

- Graphs of Polynomials: As x goes to positive and negative infinity all even degree polynomials look similar to $y = x^2$ and all odd degree polynomials look similar $y = x^3$.



Even Degree Polynomial



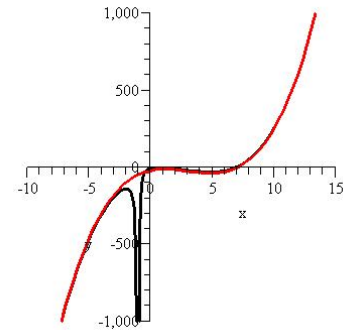
Odd Degree Polynomial

- End Behavior: The *end behavior* of a rational function is what happens to the graph of or output values of the polynomial as the value of the input x goes to positive or negative infinity.

- You find the end behavior of a rational function by dividing the numerator by the denominator and discarding the remainder. For example from above we know:

$$\frac{x^5 - 7x^4 + 3x^2 - 9}{x^2 + 2x + 1} = x^3 - 9x^2 + 17x - 22 + \frac{27x + 13}{x^2 + 2x + 1}$$

and so the end behavior is $y = x^3 - 9x^2 + 17x - 22$.



- Horizontal Asymptote: Given $y = \frac{p(x)}{q(x)}$ then y has a *horizontal asymptote* if the degree of the function in the denominator is greater than or equal to the degree in the numerator. If the degree in the denominator is greater the asymptote is $y = 0$ otherwise it is equal to the leading coefficient of the numerator divided by the leading coefficient of the denominator.
- Oblique Asymptote: An *oblique or slant asymptote* occurs when the degree of the numerator is exactly one greater than the degree of the denominator so that the end behavior of the function is a line.

- Vertical Asymptotes: A *vertical asymptote* occurs at values of x where the denominator is equal to zero and the numerator is not. Points, values of x , where the numerator and denominator are both zero prior to simplification and where the denominator is not zero after simplification are called *points of discontinuity*.