

Definitions and Theorems

Definitions

- The derivative of a function $f(x)$ at $x = a$ is
 - the slope of the tangent line to $f(x)$ at $x = a$.
 - the instantaneous rate of change of $f(x)$ at $x = a$.
- The derivative function for a continuous function $f(x)$ is

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad \text{or} \quad f'(x) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

when these limits exist.

Theorems

- If $f(x)$ is a differentiable function of x , then $f(x)$ is continuous. (Note that a function can be continuous and not differentiable, e.g. $f(x) = |x|$ at $x = 0$.)
- Mean Value Theorem for Derivatives
If $f(x)$ is differentiable on the interval $[a, b]$, then there exists $c \in [a, b]$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Key Derivative Formulas to Know

1. $\frac{d}{dx}k = 0$
2. $\frac{d}{dx}kf(x) = k\frac{d}{dx}f(x)$
3. $\frac{d}{dx}f(x) \pm g(x) = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$
4. $\frac{d}{dx}f(x) \cdot g(x) = g(x) \cdot f'(x) + f(x) \cdot g'(x)$
5. $\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{(g(x))^2}$
6. $\frac{d}{dx}f(g(x)) = f'(g(x)) \cdot g'(x)$
7. $\frac{d}{dx}x = 1$
8. $\frac{d}{dx}x^n = nx^{n-1}$
9. $\frac{d}{dx}\sin(x) = \cos(x)$
10. $\frac{d}{dx}\cos(x) = -\sin(x)$
11. $\frac{d}{dx}\tan(x) = \sec^2(x) \left(= \frac{1}{\cos^2(x)} \right)$
12. $\frac{d}{dx}\sec(x) = \sec(x)\tan(x) \left(= \frac{\sin(x)}{\cos^2(x)} \right)$
13. $\frac{d}{dx}\csc(x) = -\csc(x)\cot(x) \left(= \frac{-\cos(x)}{\sin^2(x)} \right)$
14. $\frac{d}{dx}\cot(x) = -\csc^2(x) \left(= \frac{-1}{\sin^2(x)} \right)$
15. $\frac{d}{dx}e^x = e^x$
16. $\frac{d}{dx}a^x = a^x \ln(a), a > 0$
17. $\frac{d}{dx}\ln(x) = \frac{1}{x}$
18. $\frac{d}{dx}\log_a(x) = \frac{1}{x \ln(a)}$

Other Formulas to Recognize

1. $\frac{d}{dx} \frac{1}{f(x)} = \frac{-f'(x)}{(f(x))^2}$
2. $\frac{d}{dx}f^{-1}(x) = \frac{1}{f'(f^{-1}(x))}$