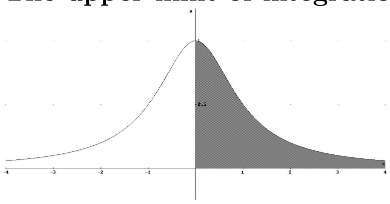
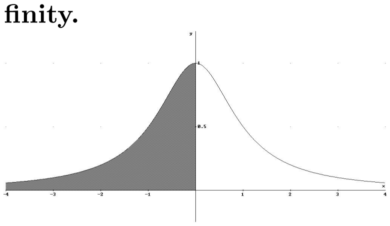
Improper Integrals

The upper limit of integration is infinity.



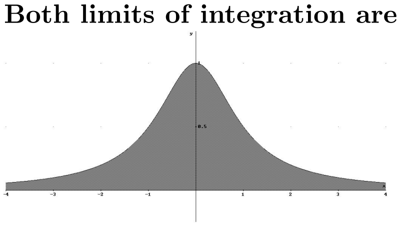
$$\begin{aligned}\int_0^{\infty} \frac{1}{x^2+1}\,dx &= \lim_{b\to\infty}\int_0^b \frac{1}{x^2+1}\,dx\\ &= \lim_{b\to\infty} \arctan(x)\Big|_0^b\\ &= \lim_{b\to\infty} \arctan(b)-\arctan(0)\\ &= \frac{\pi}{2}\end{aligned}$$

The lower limit of integration is negative infinity.



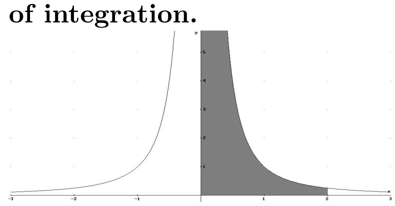
$$\begin{aligned}\int_{-\infty}^{0} \frac{1}{x^2+1}\,dx &= \lim_{a\to-\infty}\int_a^0 \frac{1}{x^2+1}\,dx\\ &= \lim_{a\to-\infty} \arctan(x)\Big|_a^0\\ &= \lim_{a\to-\infty} \arctan(0)-\arctan(a)\\ &= \frac{\pi}{2}\end{aligned}$$

Both limits of integration are infinite.



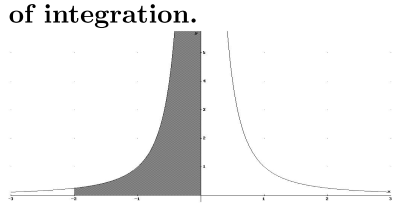
$$\begin{aligned}\int_{-\infty}^{\infty} \frac{1}{x^2+1}\,dx &= \int_{-\infty}^0 \frac{1}{x^2+1}\,dx + \int_0^{\infty} \frac{1}{x^2+1}\,dx\\ &= \lim_{a\to-\infty}\int_a^0 \frac{1}{x^2+1}\,dx + \lim_{b\to\infty}\int_0^b \frac{1}{x^2+1}\,dx\\ &= \frac{\pi}{2}+\frac{\pi}{2}\\ &= \pi\end{aligned}$$

The integrand is undefined at the lower limit of integration.



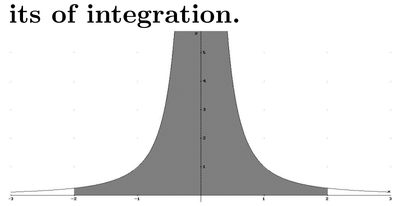
$$\begin{aligned}\int_0^2 \frac{1}{x^2}\,dx &= \lim_{a\to0^+}\int_a^2 \frac{1}{x^2}\,dx\\ &= \lim_{a\to0^+} \frac{-1}{x}\Big|_a^2\\ &= \lim_{a\to0^+} \frac{-1}{2}+\frac{1}{a}\\ &= \infty\end{aligned}$$

The integrand is undefined at the upper limit of integration.



$$\begin{aligned}\int_{-2}^0 \frac{1}{x^2}\,dx &= \lim_{b\to0^-}\int_{-2}^b \frac{1}{x^2}\,dx\\ &= \lim_{b\to0^-} \frac{-1}{x}\Big|_{-2}^b\\ &= \lim_{b\to0^-} \frac{-1}{b}+\frac{1}{2}\\ &= \infty\end{aligned}$$

The integrand is undefined in between the limits of integration.



$$\begin{aligned}\int_{-2}^2 \frac{1}{x^2}\,dx &= \int_{-2}^0 \frac{1}{x^2}\,dx + \int_0^2 \frac{1}{x^2}\,dx\\ &= \lim_{b\to0^-}\int_{-2}^b \frac{1}{x^2}\,dx + \lim_{a\to0^+}\int_a^2 \frac{1}{x^2}\,dx\\ &= \infty\end{aligned}$$