Definitions and Theorems

Definitions

- The definite integral of a continuous function $f(x)$ on an interval $[a, b]$ is

$$\int_a^b f(x)\ dx\ = \lim_{\Delta x \to 0} \sum_{i=1}^{n} f(c_i)\Delta x,$$

where n is the number of partitions, c_i is a point in the i^{th} partition, and Δx is the width of the i^{th} partition. (The definite integral is the limit of Riemann Sums.)

- The indefinite integral of a continuous function $f(x)$,

$$\int f(x)\ dx\ = F(x) + c,$$

is the set of all Antiderivatives of $f(x)$.

Theorems

- Mean Value Theorem for Integrals

If $f(x)$ is continuous on the interval $[a, b]$, then there exists $c \in [a, b]$ such that

$$f(c) = \frac{1}{b-a} \int_a^b f(x)\ dx\ =\ \text{Average of } f(x) \text{ on } [a, b].$$

- Fundamental Theorem of Calculus

If $f(x)$ is continuous on the interval $[a, b]$ and

$$F(x) = \int_a^x f(t)\ dt,$$

then $\frac{d}{dx}F(x) = f(x)$ for $a \leq x \leq b$.

If $f(x)$ is continuous on the interval $[a, b]$ and $F(x)$ is an Antiderivative of $f(x)$ then

$$\int_a^b f(x)\ dx\ = F(b) - F(a).$$

Key Integral Formulas to Know

1. $\int kf(x)\ dx\ = k \int f(x)\ dx$
2. $\int f(x) \pm g(x)\ dx\ = \int f(x)\ dx\ \pm \int g(x)\ dx$
3. $\int_a^a f(x)\ dx\ = 0$
4. $\int_b^a f(x)\ dx\ = -\int_a^b f(x)\ dx$
5. $\int_a^c f(x)\ dx\ = \int_a^b f(x)\ dx\ + \int_b^c f(x)\ dx$

6. $\int k \; dx \; = kx + c$

7. $\int x^n \; dx \; = \frac{x^{n+1}}{n+1} + c$

8. $\int \cos(x) \; dx \; = \sin(x) + c$

9. $\int \sin(x) \; dx \; = -\cos(x) + c$

10. $\int e^x \; dx \; = e^x + c$

11. $\int \frac{1}{x} \; dx \; = \ln(x) + c$

Other Formulas to Recognize

1. $\int \sec^2(x) \; dx \; = \tan(x) + c$

2. $\int \sec(x)\tan(x) \; dx \; = \sec(x) + c$

3. $\int \csc^2(x) \; dx \; = -\cot(x) + c$

4. $\int \csc(x)\cot(x) \; dx \; = -\csc(x) + c$

5. $\int a^x \; dx \; = \frac{a^x}{\ln(a)} + c$