Math 182
Fall 2004
Final Exam

Basic Skills (25% of the final exam grade)

Find the derivative and then move on, do not simplify.

1. $g(x) = 5x^3 + 13x^2 + 10$
   $g'(x) =$

2. $q = -7r^4 + r^{2/3} + 17r^{-1}$
   $q' =$

3. $y = z^{-2} + \frac{1}{z^3} - \frac{2}{z}$
   $y' =$

4. $r = e^{5\theta+1}$
   $r' =$

5. $s(t) = \cos(t) \cdot \tan(t)$
   $s'(t) =$

6. $f(x) = e^x \cdot \sin(2x - 1)$
   $f'(x) =$

7. $y = \ln(2t + 1)$
   $y' =$

8. $h(t) = \frac{t^3-1}{t^4-4t+2}$
   $h'(t) =$

9. $z = (x^2 + 3x - 1)^4 \cdot (x^3 + 1)^3$
   $z' =$

10. $f(x) = (x + 13\sin(2x))^4$
$f'(x) =$

Evaluate the integral.

11. $\int t^2 + 4t + 9 \; dt =$

12. $\int x^{4/5} - x^3 + 71 \; dx =$

13. $\int \frac{1}{x^2} + \frac{1}{x} + x \; dx =$

14. $\int \left(t^2\sqrt{t} + \frac{t}{\sqrt{t}}\right) \; dt =$

15. $\int_{-2}^{2} \left(\frac{x^3}{e} - \frac{x^2}{\pi}\right) \; dx =$

16. $\int_{0}^{\pi/2} \cos(\theta) - \sin(\theta) \; d\theta =$

17. $\int_{8}^{64} \frac{1}{x^{2/3}} \; dx =$

18. Integrate the integral using substitution:

$$\int_1^5 e^{2x-2}\, dx$$

19. Integrate the integral using substitution:

$$\int x\cos(3x^2+1)\, dx$$

20. Integrate the integral using integration by parts:

$$\int te^t\, dt$$

Short Answer (35% of the final exam grade)

1. The Derivative is the ____________________ or ____________________.

2. If the second derivative is negative then the function is ____________________ and the first derivative is ____________________.

3. The Definite Integral of a continuous function on an interval is defined to be the ______________________________, which according to the fundamental theorem of calculus is equivalent to the ______________________________.

4. If a function $f(x)$ is continuous and non-negative on an interval $[a, b]$ then the definite integral is equal to the ____________________.

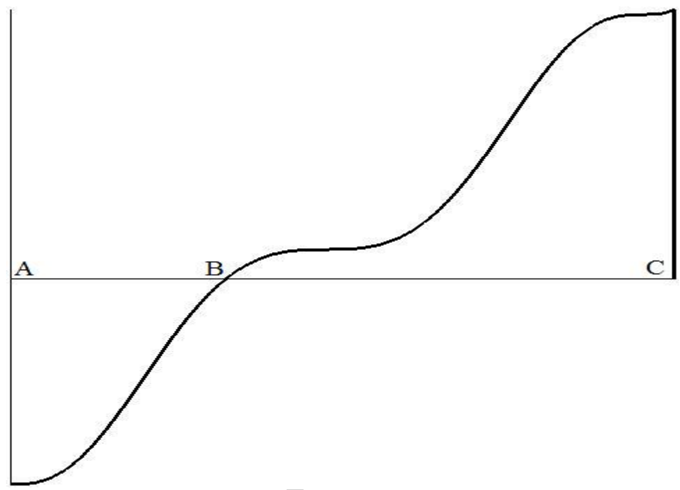

Figure 1

5. In the figure 1 is the integral of the function from A to C positive, negative, or neither?

   ____

   What about the derivative? ____________________

6. Referring to the function in figure 1 which integral is greater and why?

$$Integral\ 1 = \int_A^C f(x)\ dx \text{ or } Integral\ 2 = \left(\int_B^C f(x)\ dx - \int_A^B f(x)\ dx\right)$$

   ____

   ____

   ____

7. If a sequence is bounded it must have a

   ____________________ ____________________.

8. The sum of a series is equal to the limit of the ____________________.

Not So Basic Skills (40% of the final exam grade)

1. Revolve the region about $y = 0$ and set up the integral to find its volume.

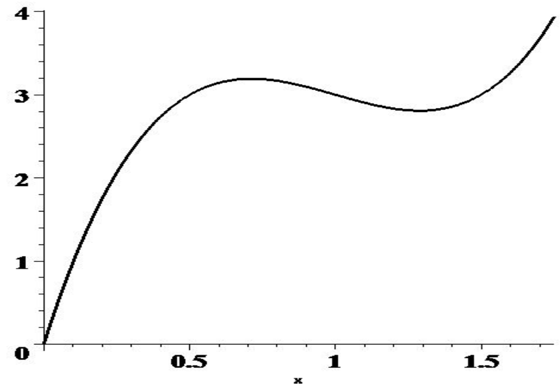

Figure 2

The region between
$y = 4x^3 - 12x^2 + 11x$, $x = 1.5$, **and** $y = 0$.

2. Set up the integral to find the arc length of the curve in the previous question on the interval from 0 to 2π.

3. Find the sum of the geometric series.
$\sum_{n=0}^{\infty} 5\left(\frac{7}{9}\right)^n$

4. Use the limit comparison or direct comparison test to determine if the series converges or diverges.
$\sum_{n=2}^{\infty} \frac{n^9}{n^{12}+15}$

Answer each of the following questions for the following 4 series.

(a) Do you think the series converges or diverges (give reasons)?

(b) Which test is appropriate for this series?

5. $\sum_{n=0}^{\infty} \frac{n^7}{n^8+37}$

6. $\sum_{n=0}^{\infty} (-1)^n \frac{n^3-2}{(n^4+9)^2}$

7. $\sum_{n=0}^{\infty} \frac{n!}{n^{1972}+32^n}$

8. $\sum_{n=0}^{\infty} \frac{2^n}{n^n+7}$

Answer three of the last four problems, the fourth you can do for extra credit.

9. For each of the following questions let $f(x) = \sqrt[5]{5x-9}$.

   (a) Find the 1^{st}, 2^{nd}, and 3^{rd} derivatives of $f(x)$ and evaluate them at $x = 2$. (Simplify)

   $f(x) = \sqrt[5]{5x-9}$ $\qquad$ $f(2) =$

   $f'(x) =$ $\qquad$ $f'(2) =$

   $f''(x) =$ $\qquad$ $f''(2) =$

   $f'''(x) =$ $\qquad$ $f'''(2) =$

   (b) Find, using derivatives, the Taylor polynomial of degree 2 for $f(x)$ centered at $a = 2$.

   (c) Find, using derivatives, the Taylor polynomial of degree 3 for $f(x)$ centered at $a = 2$.

   (d) Use each of the above approximations to get an estimate for $f(1.9)$.

10. Answer the following questions about the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n}(x-1)^n$$

(a) What test will we use to find the center and radius of convergence?

(b) Where is the center of the series?

(c) What is the radius of convergence for the given series?

(d) What is the interval of convergence?

11. A Binomial series has the form

$$(1+x)^p = 1 + px + \frac{p(p-1)}{2!}x^2 + \frac{p(p-1)(p-2)}{3!}x^3 + \frac{p(p-1)(p-2)(p-3)}{4!}x^4 + \cdots$$

Using this find the first six terms of the series for $g(x) = (1+x)^{2/9}$

And, using the series you just found find the first six terms of the series for $h(x) = (1+3x)^{2/9}$

and the first 5 terms of the series for $g'(x) = \frac{2}{9}(1+x)^{-7/9}$

12. If $f(x) = \frac{1}{x+2}$, then the n^{th} derivative of $f(x)$ is

$$f^{(n)}(x) = \frac{(-1)^n \; n!}{(x+2)^{n+1}} \quad \text{for} \quad n \geq 1.$$

Use this to answer each of the following questions. All of the Taylor Polynomials are centered at -1.

(a) What is the upper bound for the error of the 2^{nd} degree Taylor polynomial on the interval $\left[-1, \frac{-3}{4}\right]$?(Hint: Look at the absolute value of the third derivative, what does its graph look like on this interval?)

(b) What is the upper bound for the error of the 3^{rd} degree Taylor polynomial on the interval $\left[-1, \frac{-3}{4}\right]$?

(c) What is the upper bound for the error of the n^{th} degree Taylor polynomial on the interval $\left[-1, \frac{-3}{4}\right]$?