Test	Description	Comment
k^{th} term test	$\sum_{k=1}^{\infty} a_k$ diverges if $\lim\limits_{k\to\infty} a_k \neq 0$.	This test gives no information about the convergence of the series.
Direct Comparison Test (for convergence)	$\sum_{k=1}^{\infty} a_k$ converges if $0 \leq a_k \leq b_k$ and $\sum_{k=1}^{\infty} b_k$ converges.	The a_k and b_k must all be positive, and you must know the convergence of the series $\sum_{k=1}^{\infty} b_k$.
Direct Comparison Test (for divergence)	$\sum_{k=1}^{\infty} a_k$ diverges if $0 \leq b_k \leq a_k$ and $\sum_{k=1}^{\infty} b_k$ diverges.	The a_k and b_k must all be positive, and you must know the divergence of the series $\sum_{k=1}^{\infty} b_k$.
Limit Comparison Test	$\sum_{k=1}^{\infty} a_k$ converges (diverges) if $\sum_{k=1}^{\infty} b_k$ converges (diverges) and $\lim\limits_{k\to\infty} (\frac{a_k}{b_k}) = L > 0$.	The a_k and b_k must all be positive, and you must know the convergence or the divergence of the series $\sum_{k=1}^{\infty} b_k$. Note, if $L = 0(\infty)$ and $\sum_{k=1}^{\infty} b_k$ converges (diverges) then $\sum_{k=1}^{\infty} a_k$ converges (diverges).
Integral Test for series with positive terms	$\sum_{k=1}^{\infty} a_k$ converges (diverges) if for $f(k) = a_k$, $a_k \geq 0$, f continuous and non-increasing, $\int_1^{\infty} f(x)dx$ converges (diverges).	Good to use if f is easy to integrate.
Ratio Test for series with positive terms	$\sum_{k=1}^{\infty} a_k$ 1)converges if $\lim\limits_{k\to\infty} (\frac{a_{k+1}}{a_k}) < 1$, 2)diverges if $\lim\limits_{k\to\infty} (\frac{a_{k+1}}{a_k}) > 1$, 3)undetermined if $\lim\limits_{k\to\infty} (\frac{a_{k+1}}{a_k}) = 1$,	Good to use if a_k consists of factorials or k^{th} powers.
k^{th} root test for series with positive terms	$\sum_{k=1}^{\infty} a_k$ 1)converges if $\lim\limits_{k\to\infty} (\sqrt[k]{a_k}) < 1$, 2)diverges if $\lim\limits_{k\to\infty} (\sqrt[k]{a_k}) > 1$, 3)undetermined if $\lim\limits_{k\to\infty} (\sqrt[k]{a_k}) = 1$,	Good to use if a_k consists of k^{th} powers.
Alternating series test	$\sum_{k=1}^{\infty} (-1)^{k+1} u_k$ converges if $\{u_k\}$ is a non-negative, non-increasing sequence and $\lim\limits_{k\to\infty} u_k = 0$	
Absolute convergence	If $\sum_{k=1}^{\infty} \lvert a_k \rvert$ converges then $\sum_{k=1}^{\infty} a_k$ converges.	

Examples of testing for convergence.	
$\sum_{k=1}^{\infty} \frac{(-1)^{(k\,-\,1)}}{k}$	This series converges by the alternating series test. However, $\sum_{k=1}^{\infty} \frac{1}{k}$ diverges, therefore the original series converges conditionally, not absolutely.
$\sum_{k=1}^{\infty} \frac{k^k}{k!}$	The $\lim\limits_{k\to\infty} \frac{k^k}{k!}$ diverges, therefore the series diverges by the k^{th} term test.
$\sum_{k=1}^{\infty} \frac{\sin^2(k)}{k^2}$	Since, $\sin^2(k) \leq 1$ for all k, we can concluded that $\frac{\sin^2(k)}{k^2} \leq \frac{1}{k^2}$ and the series converges by direct comparison with $\sum_{k=1}^{\infty} \frac{1}{k^2}$, which is a convergent p-series.
$\sum_{k=7}^{\infty} \frac{5}{k-6}$	Since, $\frac{5}{k-6} > \frac{5}{k}$, the series diverges by direct comparison with $\sum_{k=6}^{\infty} \frac{5}{k}$, which is a multiple of the tail of the Harmonic series.
$\sum_{k=1}^{\infty} \frac{5k^3+2k+1}{k^3 k!}$	Compare $a_k = \frac{5k^3+2k+1}{k^3 k!}$ to $b_k = \frac{1}{k!}$ using the limit comparison test; $\lim\limits_{k\to\infty} \frac{a_k}{b_k} = \lim\limits_{k\to\infty} \frac{5k^3+2k+1}{k^3} = 5.$ Therefore, the series converges because the limit is neither ∞ nor 0, and because $\sum_{k=1}^{\infty} \frac{1}{k!}$ converges.
$\sum_{k=1}^{\infty} \frac{3k^2+7}{k^3+7k-1}$	The integral $\int_1^{\infty} \frac{3x^2+7}{x^3+7x-1} dx$ diverges, therefore the series diverges by the integral test.
$\sum_{k=1}^{\infty} \frac{3k^2}{k!}$	Consider, $\lim\limits_{k\to\infty} \frac{a_{k+1}}{a_k} = \lim\limits_{k\to\infty} \frac{k+1}{k^2} = 0,$ therefore, the series converges by the ratio test.
$\sum_{k=1}^{\infty} \frac{1}{k^k}$	Consider, $\lim\limits_{k\to\infty} \sqrt[k]{a_k} = \lim\limits_{k\to\infty} \sqrt[k]{\frac{1}{k^k}} = \lim\limits_{k\to\infty} \frac{1}{k} = 0,$ therefore, the series converges by the k^{th} root test.

Examples of Series

Geometric Series	$\sum_{k=1}^{\infty} ar^{k-1}$	Converges to $\frac{a}{1-r}$ if $\lvert r\rvert < 1$; diverges otherwise.
Harmonic Series	$\sum_{k=1}^{\infty} \frac{1}{k}$	Diverges.
p-Series	$\sum_{k=1}^{\infty} \frac{1}{k^p}$	Converges if $p > 1$; diverges if $p \leq 1$.
Factorial Series	$\sum_{k=1}^{\infty} \frac{1}{k!}$	Converges.
k-to-the-k Series	$\sum_{k=1}^{\infty} \frac{1}{k^k}$	Converges.
Alternating Harmonic Series	$\sum_{k=1}^{\infty} \frac{(-1)^{(k\,-\,1)}}{k}$	Converges conditionally, not absolutely.

Taylor Series

$f(x)$ centered at $a = 0$	$\sum_{k=0}^{\infty} c_k\ x^k$, where $c_k = \frac{f^{(k)}(0)}{k!}$.	May not converge everywhere
$f(x)$ centered at $a \neq 0$	$\sum_{k=0}^{\infty} c_k\ (x-a)^k$, where $c_k = \frac{f^{(k)}(a)}{k!}$.	May not converge everywhere
e^x centered at $a = 0$	$\sum_{k=0}^{\infty} \frac{x^k}{k!}$	Infinite radius of convergence
$\sin(x)$ centered at $a = 0$	$\sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$	Infinite radius of convergence
$\cos(x)$ centered at $a = 0$	$\sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}$	Infinite radius of convergence
$\arctan(x)$ centered at $a = 0$	$\sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{2k+1}$	Radius of convergence 1
$\frac{1}{1+x^2}$ centered at $a = 0$	$\sum_{k=0}^{\infty} (-x^2)^k$	Radius of convergence 1

Theorem: Error Bounds for Alternating Series

Let $S_n = \sum_{i=1}^{n} (-1)^{i-1} a_i$ be the n^{th} partial sum of an alternating series and let $S = \lim_{n\to\infty} S_n$. Suppose that $0 < a_{n+1} < a_n$ for all n and $\lim_{n\to\infty} a_n = 0$. Then

$$|S - S_n| < a_{n+1}.$$

Theorem: Bounding the Error in $\mathbf{P_n(x)}$

Suppose f and all its derivatives are continuous. If $P_n(x)$ is the n^{th} Taylor approximation to $f(x)$ about a, then

$$|E_n(x)| = |f(x) - P_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1},$$

where $M = max(|f^{(n+1)}|)$ on the interval between a and x.