Lab 4 – Related rates, etc.

My Goals:

1. To recover the Related Rates material on which you did poorly on the latest exam
2. To practice word problems
3. To use some Derive
4. To work in groups

Introduction

We had more problems with the Chapter 3 test than I expected. In this lab we will attempt to strengthen some of the skills that seemed weak on that test.

This lab comes in two parts.

In the **first part** you will find three Related Rates problems. For **each** problem you should

1. Restate the problem in your own words,
2. Draw a good picture of the problem, labeling parts appropriately,
3. Set up the necessary equations,
4. Solve the equations by hand,
5. Verify your solutions in Derive, and
6. State your solution in a complete sentence.

Do each problem on a separate page. Make these look good.

In the **second part** you will take a number of derivatives. For each example, you should take the derivatives by hand, carefully labeling each step and describing what you are doing. If the problem is a logarithmic differentiation problem (from page 258), you should do it twice, once using logarithmic differentiation and once without it. Then you should check your answers with Derive.

It is very likely that you will find that different methods give different versions of the same answer. Show that the different answers are really the same.

Do not continue any problem from one page to another page, if you can avoid it.

First part – Related Rates

Snidely Whiplash

Snidely Whiplash had tied poor Priscilla to the railroad tracks. She turns her head to see a train approaching at 100 ft/sec. She stares at a headlight 20 feet above the ground in the middle of the train front, awaiting her doom. What is the rate of change of the viewing angle when the train is 20 feet from our doomed friend? (Don't worry; Dudley DoRight will really save her.)

A Shaggy Dog Story

I just got a puppy. It seems he has discovered the bathroom. And in it is this wonderful toy hanging on the wall with a soft end dangling down that is just begging to be pulled. Which is what he is now in the habit of doing. He trots into the bathroom, grabs the end of the toilet paper, and then runs out the door with a streamer unreeling behind him.

Being a calculus geek, I saw this, and did I panic? Of course not. I thought, "If the dog runs at 2 meters per second, how quickly is the radius of the toilet paper roll decreasing?"

I decided to ask you.

Before you dive into this problem, you need a little more information. So, you ask me a few questions:

You: How thick is the toilet paper?
Me: Why do you need to know this?
You: Well if the paper were very thick then we'd expect that each turn of the roll would reduce the radius by more than if the paper were thin.
Me: This particular brand is 0.00005 meters thick (that's about 2 thousandths of an inch, and that's pretty thin.)
You: We need to know at exactly what moment we want to know this rate of change.
Me: Who do you need to know that?
You: Becaue the radius of the roll is changing at a rate which is perhaps not constant,
Me: So let's say it is at the moment when the radius of the roll is 0.05 meters (about 2 inches), or about half way down.

Sand castles

Sand falls from a conveyor belt at a rate of $10\frac{\text{cubic meters}}{\text{minute}}$. The pile of sand always forms a cone, and it is a property of this sand (called its "angle of repose") that the height of the pile always equal to 3/8 of the diameter of the base (or 3/16 of its radius.) How fast are the height and the radius changing when the pile is 5 meters high?

(Hint: Volume of a cone is given by $V = \frac{1}{3}p r^2 h$, where r is the radius of the base and h is height of the cone.)

You'll need to gather some facts

At the spring and fall equinoxes (March 21 and September 21, approximately) at the equator, the sun passes directly overhead at noon. Hence, a 50 foot flagpole standing straight up at the equator will cast its shadow straight down, and will have zero length.

That doesn't mean that the shadow isn't moving, though. It's just passing through the base of the flagpole.

Your task is to find how fast this shadow is moving at noon, at 3:00 pm and at 6:00 pm.

You know lots of facts about the motion of the earth and how fast it rotates, so I'm not going to tell them to you here. Make sure you include them in your write-up of the problem.

Second part – derivatives

Logarithmic derivatives (see directions in the Introduction)

From page 258, between number 137 and 142. Find y'.

1. $y = \dfrac{2\left(x^2+1\right)}{\sqrt{\cos 2x}}$
2. $y = \dfrac{2u2^u}{\sqrt{u^2+1}}$
3. $y = (\sin q)^{\sqrt{q}}$
4. $y = (\ln x)^{1/(\ln x)}$

Inverse trig functions

Simplify the following (from page 230, numbers 29-39)

5. $\sec\left(\tan^{-1}\dfrac{x}{2}\right)$
6. $\cos\left(\sin^{-1}\dfrac{2y}{3}\right)$
7. $\sin\left(\tan^{-1}\dfrac{x}{\sqrt{x^2+1}}\right)$
8. $\tan\left(\cos^{-1}x\right)$

Find the following derivatives. See the instructions in the Introduction. (from page 230 numbers 49-70)

9. $y = \cos^{-1}\left(\dfrac{1}{x}\right)$
10. $y = \cot^{-1}\sqrt{x}$
11. $y = \tan^{-1}(\ln x)$
12. $y = \cot^{-1}\left(\dfrac{1}{x} + \tan^{-1}x\right)$

Word problems

13. Page 231 number 75

14. Page 231 number 76.

One more

15. The two curves $y^2 = x^3$ and $2x^2 + 3y^2 = 5$ intersect at two points, (1, 1) and (1, -1).
    1. Draw both curves, showing the intersections.
    2. Find the equations of the lines through these points and *perpendicular* to the curve $y^2 = x^3$
    3. Find the equations of the lines through these points and *parallel* to the curve $2x^2 + 3y^2 = 5$.
    4. Draw more nice pictures.
    5. Comment on what this shows.

Rough grading paradigm:

Appearance:	5 points
Use of technology	5 points
Related rates	5 points each; 20 points total
Other problems	2 points each, 30 points total