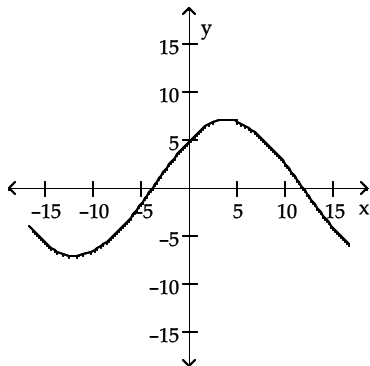


MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

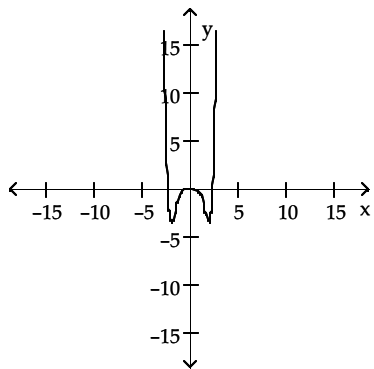
The graph of a function is given. Choose the answer that represents the graph of its derivative.

1)

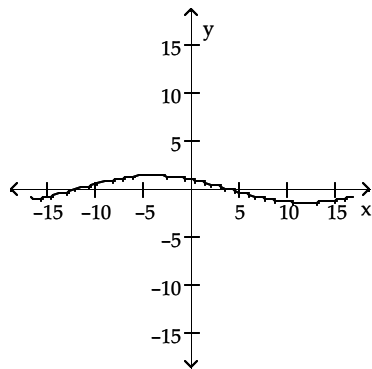
1)



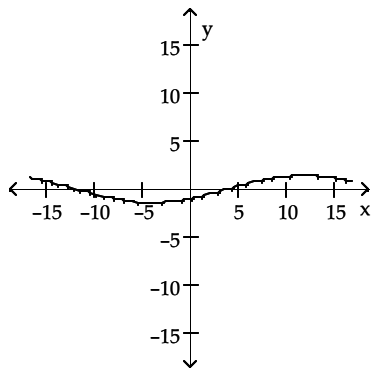
A)



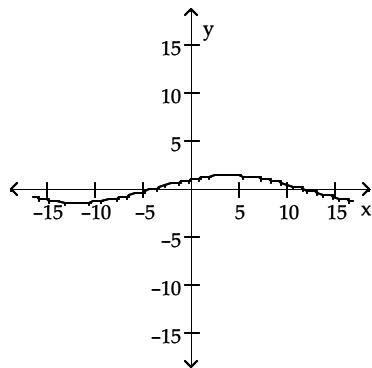
B)



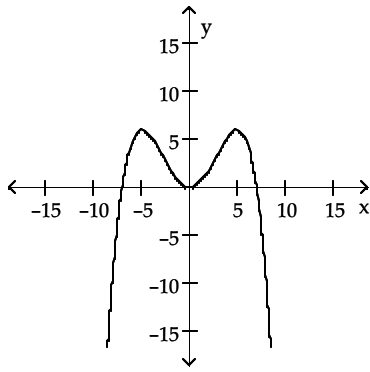
C)



D)

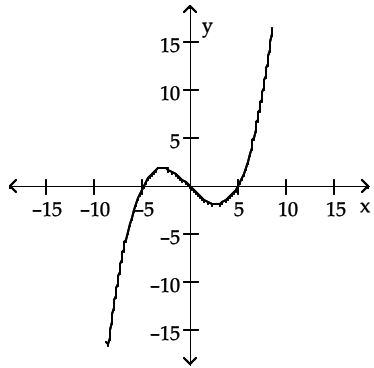


2)

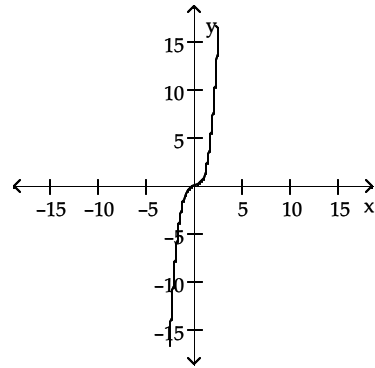


2)

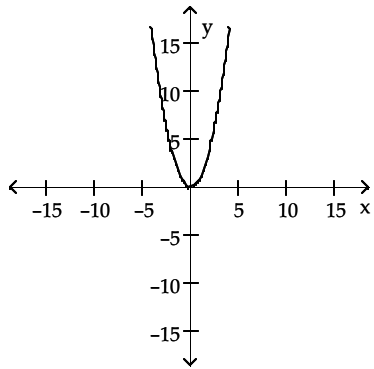
A)



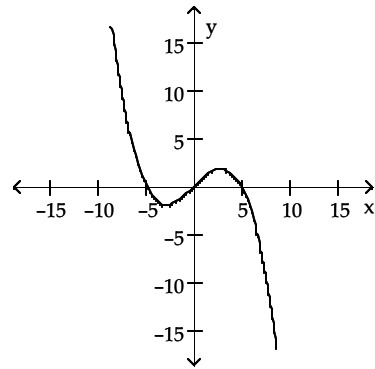
B)



C)

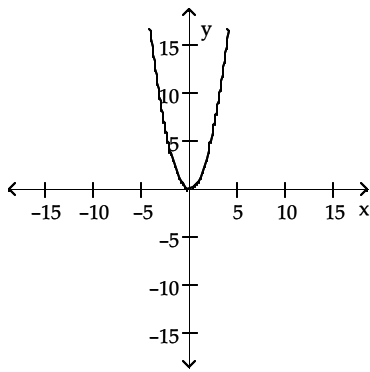


D)

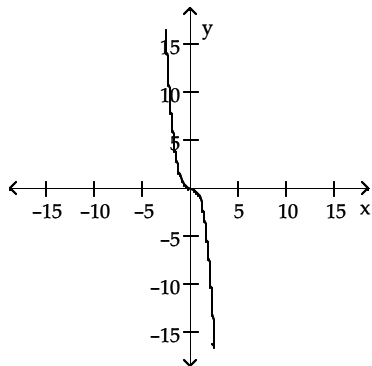


3)

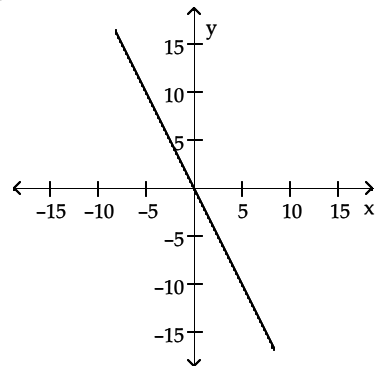
3)



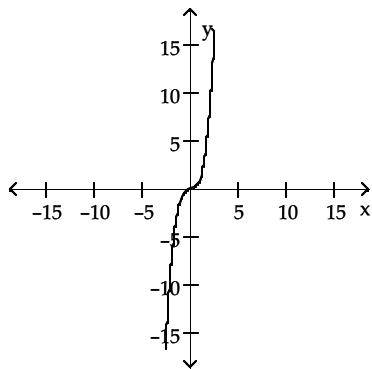
A)



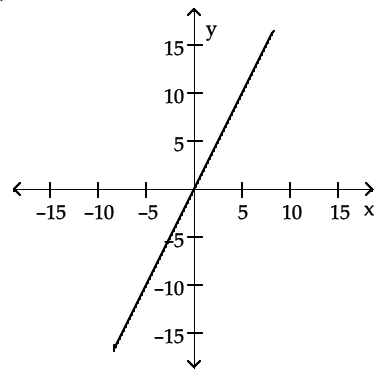
B)



C)



D)



SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Provide an appropriate response.

4) Is there any difference between finding the derivative of $f(x)$ at $x = a$ and finding the slope of the line tangent to $f(x)$ at $x = a$? Explain. 4) _____

5) Which of the following could be true if $f'(x) = -x^{-1/2}$? 5) _____

i) $f'''(x) = -\frac{3}{4}x^{-5/2}$

ii) $f''(x) = \frac{1}{2}x^{-3/2}$

iii) $f(x) = -x^{1/2}$

iv) $f(x) = -\frac{1}{2}x^{1/2}$

- 6) Derive the identity $\sec^{-1}(-x) = \pi - \sec^{-1} x$ by combining the following two equations: 6) _____
 $\cos^{-1}(-x) = \pi - \cos^{-1} x$
 $\sec^{-1} x = \cos^{-1}(1/x)$

Find a parametrization for the curve.

- 7) The upper half of the parabola $x - 1 = y^2$ 7) _____

Provide an appropriate response.

- 8) Find $d^{997}/dx^{997}(\sin x)$. 8) _____

Find the derivatives of all orders of the function.

- 9) $y = \frac{x^7}{25,200}$ 9) _____

Solve the problem.

- 10) Consider the functions $f(x) = x^2$ and $g(x) = x^3$ and their linearizations at the origin. Over some interval $-\epsilon \leq x \leq \epsilon$, the approximation error for $g(x)$ is less than the approximation error for $f(x)$ for all x within the interval. Derive a reasonable approximation for the value of ϵ . Show your work. (Hint, the absolute value of the second derivative of each function gives a measure of how quickly the slopes of the function and its linear approximation are deviating from one another.) 10) _____

Provide an appropriate response.

- 11) Assume $y = f(u)$ is a differentiable function of u and $u = g(x)$ is a differentiable function of x . If y changes m times as fast as u and u changes n times as fast as x , then y changes how many times as fast as x ? 11) _____

- 12) Find the derivative of $y = \frac{x^3 + 3x^2}{x^4}$ by using the Quotient Rule and by simplifying and then using the Power Rule for Negative Integers. Show that your answers are equivalent. 12) _____

Find a parametrization for the curve.

- 13) The ray (half line) with initial point $(-5, -9)$ that passes through the point $(-8, -6)$ 13) _____

Provide an appropriate response.

- 14) Is there anything special about the tangents to the curves $xy = 1$ and $x^2 - y^2 = 1$ at their point of intersection in the first quadrant? Explain. 14) _____

- 15) If $g(x) = 2f(x) + 3$, find $g'(4)$ given that $f'(4) = 5$. 15) _____

- 16) Find $\frac{d}{dx}\left(\frac{x^3 - 2}{x}\right)$ by using the Quotient Rule and by using the Product Rule. Show that your answers are equivalent. 16) _____

- 17) Over what intervals of x -values, if any, does the function $y = 2x^2$ increase as x increases? For what values of x , if any, is y' positive? How are your answers related? 17) _____

- 18) Graph $y = -\tan x$ and its derivative together on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Does the graph of $y = -\tan x$ appear to have a smallest slope? If so, what is it? If not, explain. 18) _____

19) Find the derivative of $y = \frac{5}{x^3}$ by using the Quotient Rule and by using the Power Rule for Negative Integers. Show that your answers are equivalent. 19) _____

20) Suppose that $u = g(x)$ is differentiable at $x = 1$, $y = f(u)$ is differentiable at $u = g(1)$, and $(f \circ g)'(1)$ is positive. What can be said about the values of $g'(1)$ and $f'(g(1))$? Explain. 20) _____

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

21) Find the points on the curve $x^2 - xy + y^2 = 12$ where the tangent is parallel to the x-axis. 21) _____
 A) $(-4, -2), (-2, 2), (2, -2), (4, 2)$ B) $(-2, -4), (2, 4)$
 C) $(4, -2), (4, 2)$ D) $(-2, -4), (-2, 2), (2, -2), (2, 4)$

Given $y = f(u)$ and $u = g(x)$, find $dy/dx = f'(g(x))g'(x)$.

22) $y = \tan u$, $u = -3x + 4$ 22) _____
 A) $\sec^2(-3x + 4)$ B) $-3 \sec(-3x + 4) \tan(-3x + 4)$
 C) $-\sec^2(-3x + 4)$ D) $-3 \sec^2(-3x + 4)$

Use implicit differentiation to find dy/dx and d^2y/dx^2 .

23) $xy - x + y = 2$ 23) _____
 A) $\frac{dy}{dx} = \frac{y+1}{x+1}; \frac{d^2y}{dx^2} = \frac{2y+2}{(x+1)^2}$ B) $\frac{dy}{dx} = -\frac{1+y}{x+1}; \frac{d^2y}{dx^2} = \frac{2y-2}{(x+1)^2}$
 C) $\frac{dy}{dx} = \frac{1-y}{1+x}; \frac{d^2y}{dx^2} = \frac{2y-2}{(x+1)^2}$ D) $\frac{dy}{dx} = -\frac{1+y}{x+1}; \frac{d^2y}{dx^2} = \frac{y+1}{(x+1)^2}$

Find the indicated derivative.

24) Find y'' if $y = 5x \sin x$. 24) _____
 A) $y'' = 5 \cos x - 10x \sin x$ B) $y'' = -5x \sin x$
 C) $y'' = -10 \cos x + 5x \sin x$ D) $y'' = 10 \cos x - 5x \sin x$

Find the formula for df^{-1}/dx .

25) $f(x) = (3 - x)^3$ 25) _____
 A) $\frac{-1}{3x^{2/3}}$ B) $-3(3 - x)^2$ C) $x^{2/3}$ D) $3 - x^{1/3}$

Find y' .

26) $y = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$ 26) _____
 A) $2x + \frac{1}{x^2}$ B) $2x + \frac{2}{x^3}$ C) $2x - \frac{1}{x^2}$ D) $2x + \frac{1}{x^3}$

Find dy .

27) $y = \sin(2x^2)$ 27) _____
 A) $4x \cos(2x^2) dx$ B) $-4 \cos(2x^2) dx$ C) $-4x \cos(2x^2) dx$ D) $4 \cos(2x^2) dx$

Use implicit differentiation to find dy/dx .

28) $\cos xy + x^5 = y^5$

A) $\frac{5x^4 - x \sin xy}{5y^4}$

B) $\frac{5x^4 + y \sin xy}{5y^4 - x \sin xy}$

C) $\frac{5x^4 + x \sin xy}{5y^4}$

D) $\frac{5x^4 - y \sin xy}{5y^4 + x \sin xy}$

28) _____

Solve the problem. Round your answer, if appropriate.

29) Water is being drained from a container which has the shape of an inverted right circular cone. The container has a radius of 5.00 inches at the top and a height of 7.00 inches. At the instant when the water in the container is 4.00 inches deep, the surface level is falling at a rate of 0.7 in./sec. Find the rate at which water is being drained from the container.

A) 18.8 in.³/s

B) 17.1 in.³/s

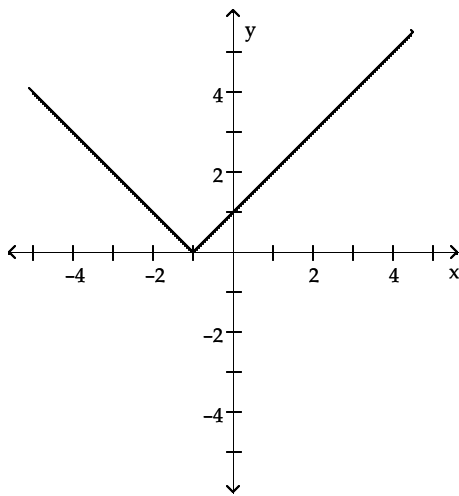
C) 18.0 in.³/s

D) 22.7 in.³/s

29) _____

The figure shows the graph of a function. At the given value of x , does the function appear to be differentiable, continuous but not differentiable, or neither continuous nor differentiable?

30) $x = -1$



30) _____

A) Differentiable

B) Continuous but not differentiable

C) Neither continuous nor differentiable

Answer Key

Testname: PRACTICE TEST CH 3 A

1) B

Objective: (3.1) Match Graph of Function with Graph of Derivative

2) D

Objective: (3.1) Match Graph of Function with Graph of Derivative

3) D

Objective: (3.1) Match Graph of Function with Graph of Derivative

4) There is no difference at all. At $x = a$, the slope of the tangent = $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = f'(x)$.

Objective: (3.1) *Know Concepts: The Derivative as a Function

5) Answers i and ii could both be true.

Objective: (3.6) *Know Concepts: Implicit Differentiation

6) $\sec^{-1}(-x) = \cos^{-1}(-1/x) = \pi - \cos^{-1}(1/x) = \pi - \sec^{-1}x$

Objective: (3.8) *Know Concepts: Inverse Trig Functions

7) Answers will vary. Possible answer: $x = y^2 - 1$, $y = t$, $t \geq 0$

Objective: (3.5) Find Parametrization for Curve

8) $\cos x$

Objective: (3.4) *Know Concepts: Derivatives of Trigonometric Functions

9) $y' = \frac{x^6}{3600}$, $y'' = \frac{x^5}{600}$, $y''' = \frac{x^4}{120}$, $y^{(4)} = \frac{x^3}{30}$, $y^{(5)} = \frac{x^2}{10}$, $y^{(6)} = \frac{x}{5}$, $y^{(7)} = \frac{1}{5}$, $y^{(n)} = 0$ for $n \geq 8$

Objective: (3.2) Find Derivatives of All Orders

10) $\varepsilon \approx \frac{1}{3}$. The slopes of $g(x)$ and its linear approximation are deviating from each other more slowly than the slopes of the

corresponding curves for $f(x)$ as long as $|g''(x)| < |f''(x)|$. Since $f''(x) = 2$ and $g''(x) = 6x$, then

$$|g''(x)| < |f''(x)| \rightarrow |6x| < 2 \text{ or } x < \left| \frac{1}{3} \right| \text{ or } -\frac{1}{3} \leq x \leq \frac{1}{3}. \text{ Thus, } \varepsilon \approx \frac{1}{3}.$$

Objective: (3.10) *Know Concepts: Linearization and Differentials

11) mn

Objective: (3.5) *Know Concepts: The Chain Rule

12) By the Quotient Rule, $\frac{dy}{dx} = \frac{x^4(3x^2 + 6x) - (x^3 + 3x^2)4x^3}{x^8} = \frac{-x^6 - 6x^5}{x^8} = -\frac{1}{x^2} - \frac{6}{x^3}$.

By the Power Rule (after simplification), $\frac{dy}{dx} = \frac{d}{dx}(x^{-1} + 3x^{-2}) = -x^{-2} - 6x^{-3} = -\frac{1}{x^2} - \frac{6}{x^3}$.

$$-\frac{1}{x^2} - \frac{6}{x^3} = -\frac{1}{x^2} - \frac{6}{x^3}$$

Objective: (3.2) *Know Concepts: Differentiation Rules

13) Answers will vary. Possible answer: $x = -5 - 3t$, $y = -9 + 3t$, $t \geq 0$

Objective: (3.5) Find Parametrization for Curve

14) The tangents are perpendicular. For the curve $xy = 1$, $\frac{dy}{dx} = -\frac{y}{x}$. For the curve $x^2 - y^2 = 1$, $\frac{dy}{dx} = \frac{x}{y}$. The two derivatives

are negative reciprocals, and thus the tangents are perpendicular.

Objective: (3.6) *Know Concepts: Implicit Differentiation

15) $g'(4) = 10$

Objective: (3.1) *Know Concepts: The Derivative as a Function

Answer Key

Testname: PRACTICE TEST CH 3 A

16) By the Quotient Rule, $\frac{d}{dx}\left(\frac{x^3 - 2}{x}\right) = \frac{x \cdot (3x^2) - (x^3 - 2) \cdot 1}{x^2} = \frac{2x^3 + 2}{x^2}$.

By the Product Rule, $\frac{d}{dx}\left(\frac{x^3 - 2}{x}\right) = \frac{d}{dx}((x^3 - 2)x^{-1}) = (x^3 - 2)(-x^{-2}) + x^{-1}(3x^2) = 2x + 2x^{-2} = \frac{2x^3 + 2}{x^2}$.

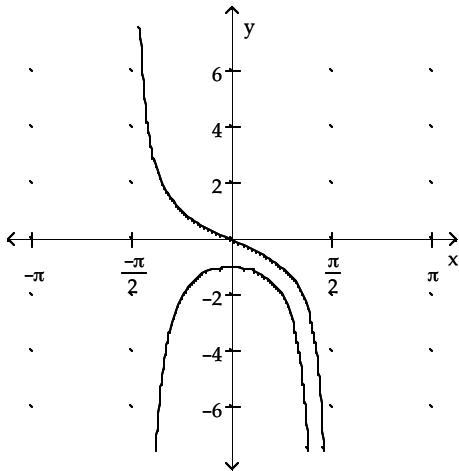
$$\frac{2x^3 + 2}{x^2} = \frac{2x^3 + 2}{x^2}$$

Objective: (3.2) *Know Concepts: Differentiation Rules

- 17) The function $y = 2x^2$ increases (as x increases) over the interval $0 < x < \infty$. Its derivative $y' = 4x$ is positive for $x > 0$. The function increases (as x increases) wherever its derivative is positive.

Objective: (3.1) *Know Concepts: The Derivative as a Function

18)



No, the graph of $y = -\tan x$ does not appear to have a smallest slope. The slope of the graph of $y = -\tan x$ decreases without bound as x approaches $\pi/2$ or $-\pi/2$.

Objective: (3.4) *Know Concepts: Derivatives of Trigonometric Functions

19) By the Quotient Rule, $\frac{dy}{dx} = \frac{x^3 \cdot 0 - 5 \cdot 3x^2}{x^6} = \frac{-15x^2}{x^6} = -\frac{15}{x^4}$.

By the Power Rule, $\frac{dy}{dx} = \frac{d}{dx}(5x^{-3}) = -15x^{-4} = -\frac{15}{x^4}$.

$$-\frac{15}{x^4} = -\frac{15}{x^4}$$

Objective: (3.2) *Know Concepts: Differentiation Rules

- 20) The values of $g'(1)$ and $f'(g(1))$ must have the same sign, since $(f \circ g)'(1) = f'(g(1))g'(1) > 0$.

Objective: (3.5) *Know Concepts: The Chain Rule

21) B

Objective: (3.6) Solve Apps: Slopes, Tangents, and Normals

22) D

Objective: (3.5) Find dy/dx Given $y = f(u)$ and $u = g(x)$

23) C

Objective: (3.6) Use Implicit Differentiation to Find First and Second Derivatives

Answer Key

Testname: PRACTICE TEST CH 3 A

- 24) D
Objective: (3.4) Find Higher-Order Derivative of Trigonometric Function
- 25) A
Objective: (3.7) Find Formula for Derivative of Inverse
- 26) B
Objective: (3.2) Find Derivative of Product
- 27) A
Objective: (3.10) Find Differential
- 28) D
Objective: (3.6) Use Implicit Differentiation to Find Derivative
- 29) C
Objective: (3.9) Solve Apps: Related Rates II
- 30) B
Objective: (3.1) Determine if Function Differentiable/Continuous at Point