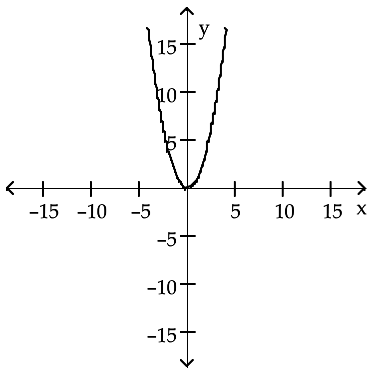
Exam

Name____________________________________

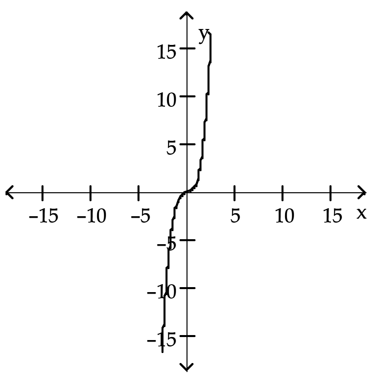
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

The graph of a function is given. Choose the answer that represents the graph of its derivative.

1) 1)



A)



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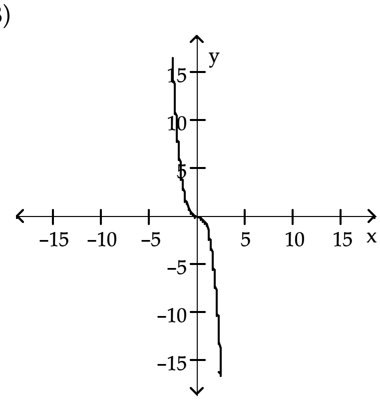

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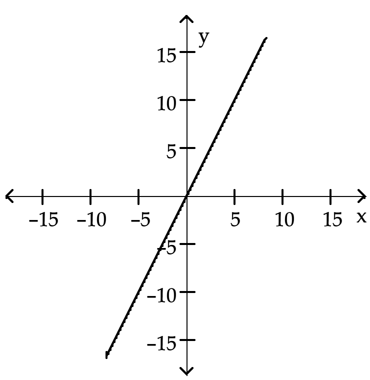

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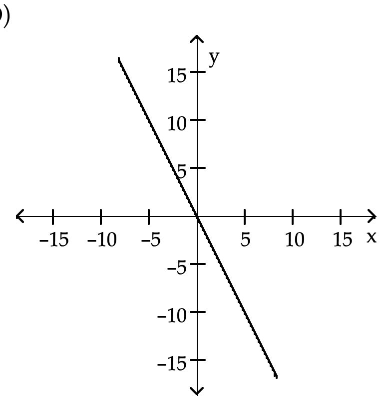

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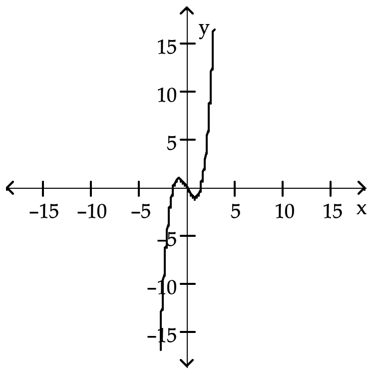
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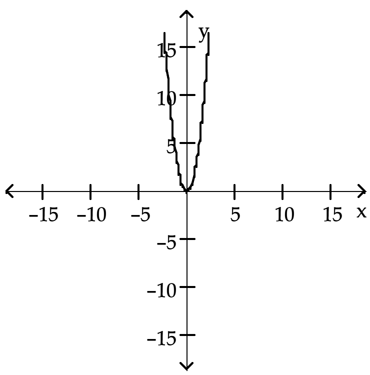
A)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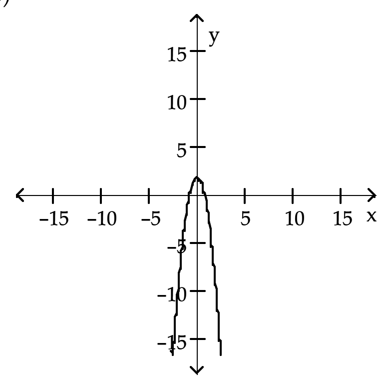
B)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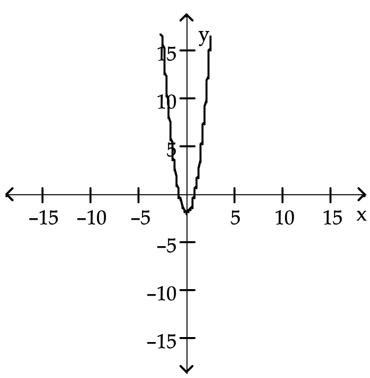
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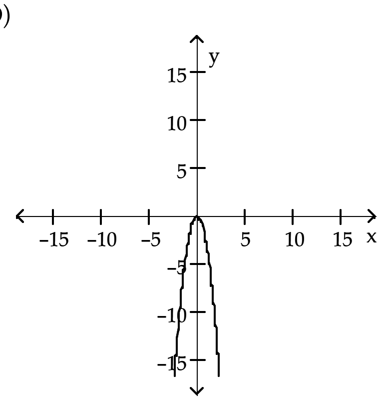
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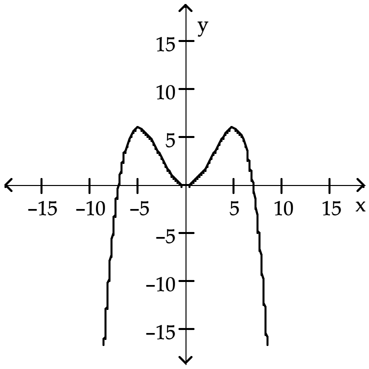
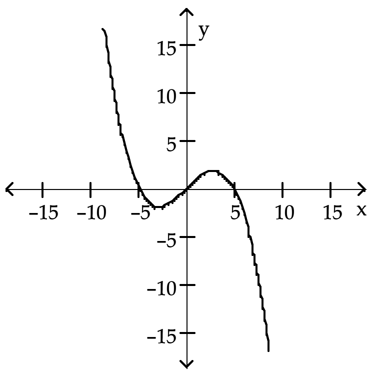
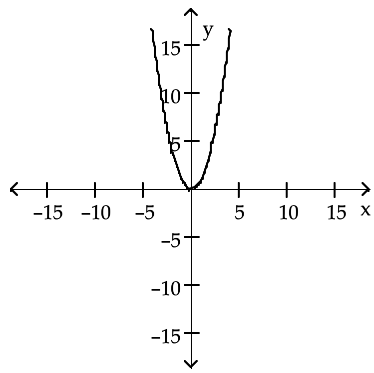
3) 3)



A)



B)



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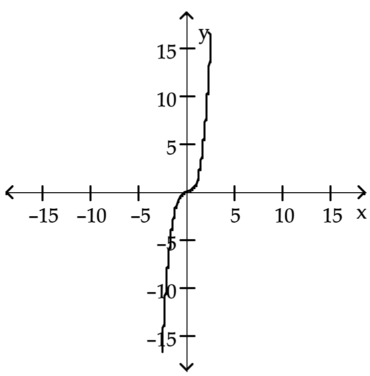
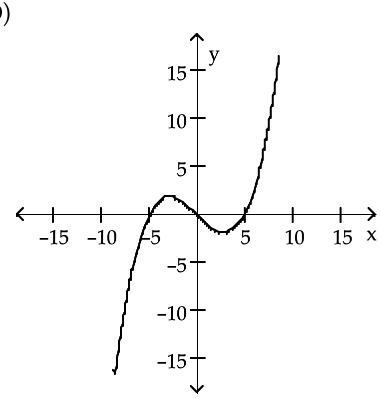


D)



Use logarithmic differentiation to find the derivative of y.

4) $y = \sqrt{x(x+4)}$ 4) ________

A) $\left(\frac{1}{2}\right)\left(\frac{1}{x} + \frac{1}{x+4}\right)$

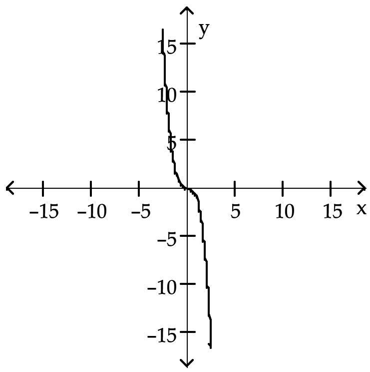
B) $\sqrt{x(x+4)}(2x+4)$

C) $\left(\frac{\sqrt{x(x+4)}}{2}\right)\left(\frac{1}{x} + \frac{1}{x+4}\right)$

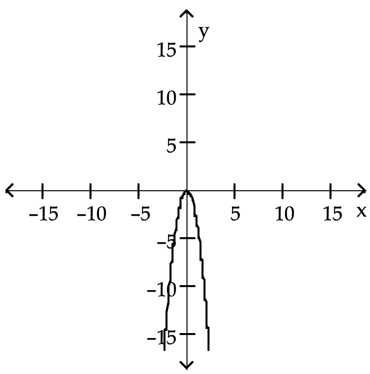
D) $\left(\frac{\ln x + \ln(x+4)}{2}\right)$

The graph of a function is given. Choose the answer that represents the graph of its derivative.

5) 5)



A)



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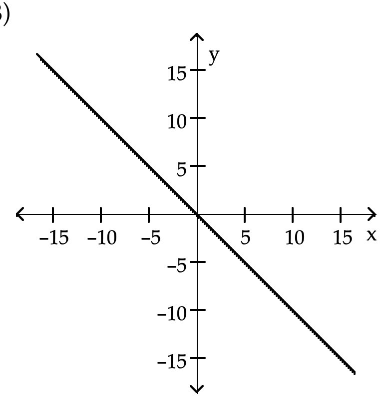

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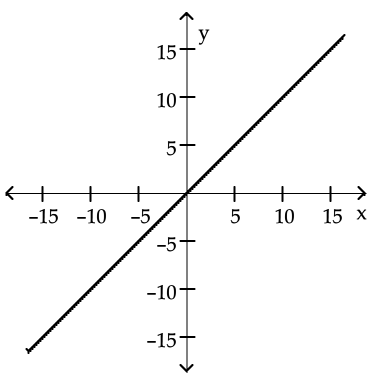

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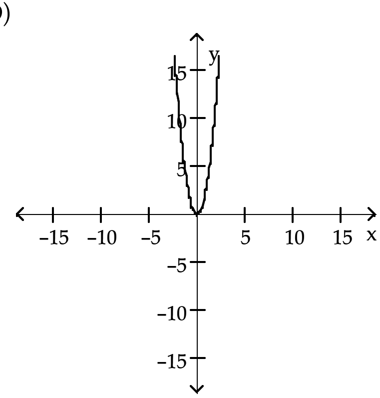
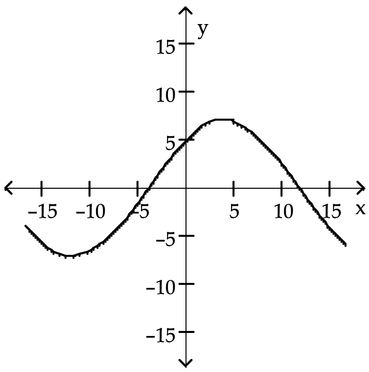


6) 6)



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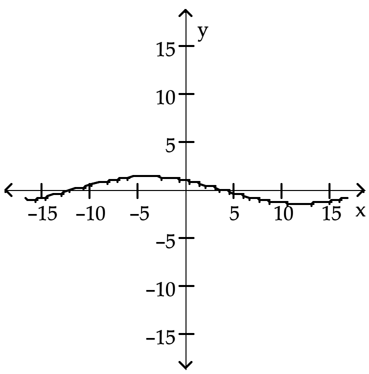

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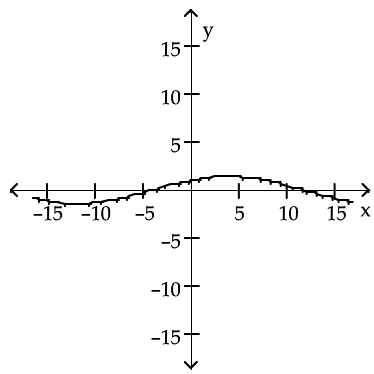

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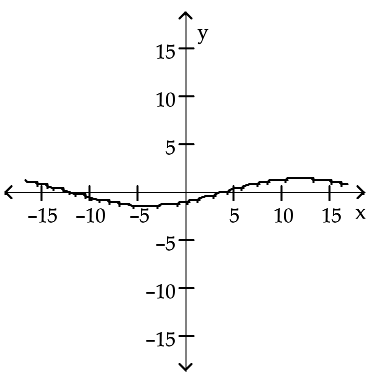

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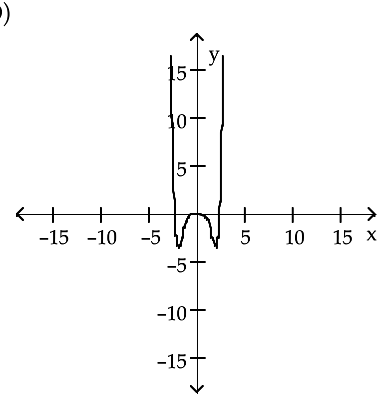
D)



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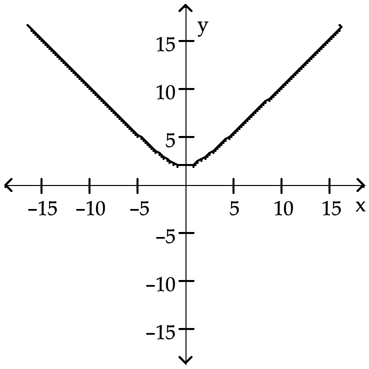
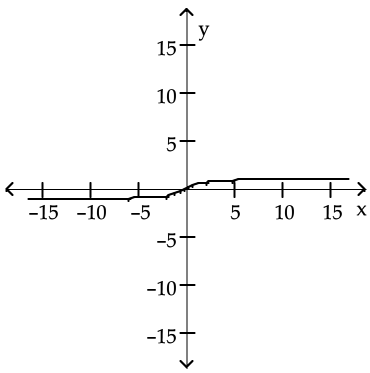
7) 7)



A)



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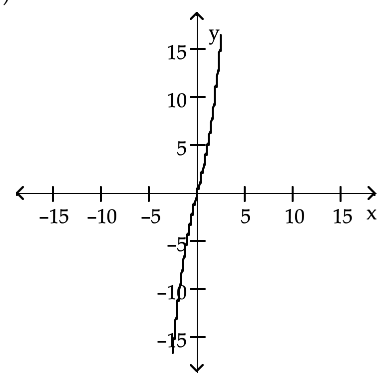

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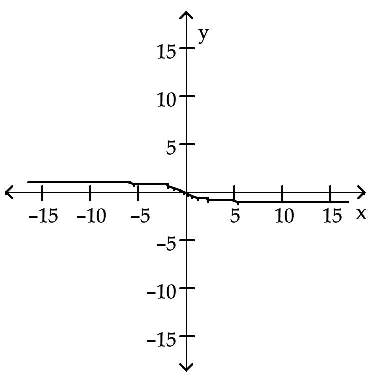
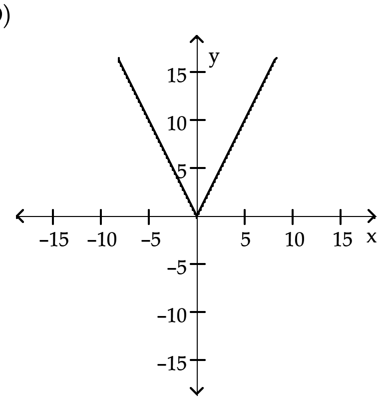


D)



SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Provide an appropriate response.

8) Is there any difference between finding the derivative of f(x) at x = a and finding the slope of the line tangent to f(x) at x = a? Explain. 8) _______________

Solve the problem.

9) Let $Q(x) = b_0 + b_1(x - a) + b_2(x - a)^2$ be a quadratic approximation to f(x) at x = a with the properties: 9) ______________

i. $Q(a) = f(a)$

ii. $Q'(a) = f'(a)$

iii. $Q''(a) = f''(a)$

(a) Find the quadratic approximation to $f(x) = \frac{1}{3 + x}$ at x = 0.

(b) Do you expect the quadratic approximation to be more or less accurate than the linearization? Give reasons for your answer.

Provide an appropriate response.

10) Graph $f(x) = \cos^{-1} \frac{x}{\sqrt{x^2 + 1}}$ and $g(x) = \tan^{-1} \frac{1}{x}$. Explain why the graph looks like it does. 10) ______________

11) Find d^{998}/dx^{998} (cos x). 11) ______________

12) If $g(x) = - f(x) - 3$, find $g'(4)$ given that $f'(4) = 5$. 12) ______________

13) Does the curve $y = x^3 + 4x - 10$ have a tangent whose slope is −2? If so, find an equation for the line and the point of tangency. If not, why not? 13) ______________

Solve the problem.

14) Consider the functions $f(x) = x^2$ and $g(x) = x^3$ and their linearizations at the origin. Over some interval $-\varepsilon \le x \le \varepsilon$, the approximation error for g(x) is less than the approximation error for f(x) for all x within the interval. Derive a reasonable approximation for the value of ε. Show your work. (Hint, the absolute value of the second derivative of each function gives a measure of how quickly the slopes of the function and its linear approximation are deviating from one another.) 14) ______________

Find the derivatives of all orders of the function.

15) $y = \frac{8}{3}x^3 + \frac{5}{2}x^2 + 5x - 16$ 15) ______________

Provide an appropriate response.

16) Find a value of c that will make 16) ______________

$$f(x) = \begin{cases} \frac{\sin^2 4x}{x^2}, & x \neq 0 \\ c, & x = 0 \end{cases}$$

continuous at x = 0.

17) Is there anything special about the tangents to the curves xy = 1 and $x^2 - y^2 = 1$ at their point of intersection in the first quadrant? Explain. 17) ______________

18) Find dy/dt when x = 5 if $y = 2x^2 - 6x + 7$ and dx/dt = 1/2. 18) ______________

19) Consider the graphs of $y = \cos^{-1} x$ and $y = \sin^{-1} x$. Does it make sense that the derivatives of these functions are opposites? Explain. 19) ______________

Solve the problem.

20) For functions of the form $y = ax^n$, show that the relative uncertainty $\left|\frac{dy}{y}\right|$ in the dependent variable y is always $|n|$ times the relative uncertainty $\left|\frac{dx}{x}\right|$ in the independent variable x. 20) ______

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the derivative of the function.

21) $y = 10\sqrt{x + 7}$ 21) ______

A) $\frac{dy}{dx} = \frac{5}{\sqrt{x + 7}}$
B) $\frac{dy}{dx} = -\frac{5}{\sqrt{x + 7}}$
C) $\frac{dy}{dx} = -\frac{5}{(x + 7)^{3/2}}$
D) $\frac{dy}{dx} = \frac{1}{2\sqrt{x + 7}}$

Calculate the derivative of the function. Then find the value of the derivative as specified.

22) $f(x) = 5x + 9$; $f'(2)$ 22) ______

A) $f'(x) = 0$; $f'(2) = 0$
B) $f'(x) = 9$; $f'(2) = 9$
C) $f'(x) = 5x$; $f'(2) = 10$
D) $f'(x) = 5$; $f'(2) = 5$

Suppose that the functions f and g and their derivatives with respect to x have the following values at the given values of x. Find the derivative with respect to x of the given combination at the given value of x.

23) 23) ______

x	f(x)	g(x)	f′(x)	g′(x)
3	1	16	6	5
4	−3	3	5	−6

$\sqrt{g(x)}$, $x = 3$

A) $\frac{5}{8}$
B) $\frac{1}{8}$
C) $-\frac{1}{2\sqrt{5}}$
D) $\frac{1}{2\sqrt{5}}$

Find the derivative of the function.

24) $h(x) = \left(\frac{\cos x}{1 + \sin x}\right)^4$ 24) ______

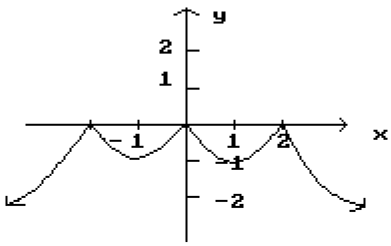
A) $h'(x) = \frac{-4\cos^3 x}{(1 + \sin x)^4}$
B) $h'(x) = \left(-\frac{4\sin x}{\cos x}\right)\left(\frac{\cos x}{1 + \sin x}\right)^3$
C) $h'(x) = 4\left(\frac{\cos x}{1 + \sin x}\right)^3$
D) $h'(x) = -4\left(\frac{\sin x}{\cos x}\right)^3$

25) $y = x^5e^{-x}$ 25) ______

A) $\frac{dy}{dx} = 5x^6e^{-x} + x^5e^{-x}$
B) $\frac{dy}{dx} = 5x^4e^{-x} - x^5e^{-x}$
C) $\frac{dy}{dx} = 5x^6e^{-x} - x^5e^{-x}$
D) $\frac{dy}{dx} = 5x^4e^{-x} + x^5e^{-x}$

Given the graph of f, find any values of x at which f' is not defined.

26) 26) ________



A) $x = 0$ B) $x = -2, 0, 2$

C) $x = -2, 2$ D) Defined for all values of x

Solve the problem.

27) A charged particle of mass m and charge q moving in an electric field E has an acceleration a given by 27) ________

$$a = \frac{qE}{m},$$

where q and E are constants. Find $\frac{da}{dm}$.

A) $\frac{da}{dm} = -\frac{m}{qE}$ B) $\frac{da}{dm} = \frac{qE}{m^2}$ C) $\frac{da}{dm} = qEm$ D) $\frac{da}{dm} = -\frac{qE}{m^2}$

Evaluate exactly.

28) $\sin\left(\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$ 28) ________

A) $\frac{-\sqrt{3}}{2}$ B) 1 C) $\frac{-1}{2}$ D) $\frac{1}{2}$

Find the derivative of the function.

29) $g(x) = x(x^9 + 8)^{1/3}$ 29) ________

A) $g'(x) = \frac{4x^9 + 8}{(x^9 + 8)^{2/3}}$ B) $g'(x) = \frac{4x^9}{(x^9 + 8)^{1/3}}$

C) $g'(x) = \frac{3x^9 + x + 24}{3(x^9 + 8)^{2/3}}$ D) $g'(x) = \frac{3x^8}{(x^9 + 8)^{2/3}}$

Find the derivative of y with respect to x, t, or θ, as appropriate.

30) $y = \ln 7x$ 30) ________

A) $-\frac{1}{x}$ B) $\frac{1}{x}$ C) $\frac{1}{7x}$ D) $-\frac{1}{7x}$

1) C
ID: TCALT11W 3.1.5-1
Diff: 0
Objective: (3.1) Match Graph of Function with Graph of Derivative

2) C
ID: TCALT11W 3.1.5-2
Diff: 0
Objective: (3.1) Match Graph of Function with Graph of Derivative

3) A
ID: TCALT11W 3.1.5-6
Diff: 0
Objective: (3.1) Match Graph of Function with Graph of Derivative

4) C
ID: TCALT11W 3.7.3-1
Diff: 0
Objective: (3.7) Perform Logarithmic Differentiation I

5) A
ID: TCALT11W 3.1.5-5
Diff: 0
Objective: (3.1) Match Graph of Function with Graph of Derivative

6) A
ID: TCALT11W 3.1.5-4
Diff: 0
Objective: (3.1) Match Graph of Function with Graph of Derivative

7) A
ID: TCALT11W 3.1.5-3
Diff: 0
Objective: (3.1) Match Graph of Function with Graph of Derivative

8) There is no difference at all. At x = a, the slope of the tangent = $\lim\limits_{x \to a} \dfrac{f(x) - f(a)}{x - a} = f'(x)$.

ID: TCALT11W 3.1.10-10
Diff: 0
Objective: (3.1) *Know Concepts: The Derivative as a Function

9) (a) $Q(x) = \dfrac{1}{3} - \dfrac{x}{9} + \dfrac{x^2}{27}$

(b) I expect the quadratic approximation to be more accurate than the linearization. The linearization accounts for how the value of f(x) is changing at the point x = a, but the quadratic approximation also accounts for the change in this change by incorporating the second derivative into the approximation.
ID: TCALT11W 3.10.8-1
Diff: 0
Objective: (3.10) *Know Concepts: Linearization and Differentials

10) When x is positive these graphs are identical because they are both giving the same angle. $\cos \theta = \frac{x}{\sqrt{x^2+1}} \leftrightarrow \tan \theta = \frac{1}{x}$. When x is negative both functions are still referring to the same angle. However, inverse cosine gives values between $\pi/2$ and π while inverse tangent gives values between $-\pi/2$ and 0.
ID: TCALT11W 3.8.6-6
Diff: 0
Objective: (3.8) *Know Concepts: Inverse Trig Functions

11) $-\cos x$
ID: TCALT11W 3.4.6-2
Diff: 0
Objective: (3.4) *Know Concepts: Derivatives of Trigonometric Functions

12) $g'(4) = -5$
ID: TCALT11W 3.1.10-9
Diff: 0
Objective: (3.1) *Know Concepts: The Derivative as a Function

13) The curve has no tangent whose slope is −2. The derivative of the curve, $y' = 3x^2 + 4$, is always positive and thus never equals −2.
ID: TCALT11W 3.1.10-6
Diff: 0
Objective: (3.1) *Know Concepts: The Derivative as a Function

14) $\varepsilon \approx \frac{1}{3}$. The slopes of g(x) and its linear approximation are deviating from each other more slowly than the slopes of the corresponding curves for f(x) as long as $|g''(x)| < |f''(x)|$. Since $f''(x) = 2$ and $g''(x) = 6x$, then $|g''(x)| < |f''(x)| \rightarrow |6x| < 2$ or $x < \left|\frac{1}{3}\right|$ or $-\frac{1}{3} \le x \le \frac{1}{3}$. Thus, $\varepsilon \approx \frac{1}{3}$.
ID: TCALT11W 3.10.8-3
Diff: 0
Objective: (3.10) *Know Concepts: Linearization and Differentials

15) $y' = 8x^2 + 5x + 5$, $y'' = 16x + 5$, $y''' = 16$, $y^{(n)} = 0$ for $n \ge 4$
ID: TCALT11W 3.2.5-1
Diff: 0
Objective: (3.2) Find Derivatives of All Orders

16) $c = 16$
ID: TCALT11W 3.4.6-1
Diff: 0
Objective: (3.4) *Know Concepts: Derivatives of Trigonometric Functions

17) The tangents are perpendicular. For the curve $xy = 1$, $\frac{dy}{dx} = -\frac{y}{x}$. For the curve $x^2 - y^2 = 1$, $\frac{dy}{dx} = \frac{x}{y}$. The two derivatives are negative reciprocals, and thus the tangents are perpendicular.
ID: TCALT11W 3.6.7-6
Diff: 0
Objective: (3.6) *Know Concepts: Implicit Differentiation

18) 7
ID: TCALT11W 3.5.12-2
Diff: 0
Objective: (3.5) *Know Concepts: The Chain Rule

19) Yes, They both have domains $-1 \le x \le 1$. They have the same basic shape with opposite slopes. Since the slopes are opposites the derivatives will be opposites.
ID: TCALT11W 3.8.6-4
Diff: 0
Objective: (3.8) *Know Concepts: Inverse Trig Functions

20) $dy = nax^{n-1}\,dx$, therefore $\frac{dy}{y} = \frac{nax^{n-1}\,dx}{ax^n} = n\frac{dx}{x}$.

Therefore, $\left|\frac{dy}{y}\right| = |n|\left|\frac{dx}{x}\right|$. Thus, the relative uncertainty in the dependent variable y is always $|n|$ times the relative uncertainty in the independent variable x.
ID: TCALT11W 3.10.8-2
Diff: 0
Objective: (3.10) *Know Concepts: Linearization and Differentials

21) A
ID: TCALT11W 3.6.1-3
Diff: 0
Objective: (3.6) Find Derivative of Rational Power

22) D
ID: TCALT11W 3.1.1-1
Diff: 0
Objective: (3.1) Find Derivative and Evaluate at Point

23) A
ID: TCALT11W 3.5.6-2
Diff: 0
Objective: (3.5) Apply Chain Rule Given f, g, f ', g '

24) A
ID: TCALT11W 3.5.3-7
Diff: 0
Objective: (3.5) Find Derivative Using Chain Rule

25) B
ID: TCALT11W 3.2.4-10
Diff: 0
Objective: (3.2) Find Derivative of Quotient

26) B
ID: TCALT11W 3.1.6-8
Diff: 0
Objective: (3.1) Find Any Values at Which Derivative is Not Defined

27) D
ID: TCALT11W 3.2.9-5
Diff: 0
Objective: (3.2) Solve Apps: Differentiation Rules

28) D
ID: TCALT11W 3.8.2-3
Diff: 0
Objective: (3.8) Evaluate Expression I

29) A
ID: TCALT11W 3.6.1-6
Diff: 0
Objective: (3.6) Find Derivative of Rational Power

30) B
ID: TCALT11W 3.7.2-1
Diff: 0
Objective: (3.7) Find Derivative of Logarithmic Function