

Practice Exam

Math 181 – Calculus I
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December 13, 2005

1. Problems are worth 10 points each, unless otherwise indicated. Parts of problems are worth parts of that.
2. Pay close attention to the instructions.
3. On problems that require calculations, show your work on the page. Use additional paper if necessary.
4. Where a blank is provided for your answer, use it.

1. Using limits, find the slope of the function $f(x) = x^3 - 3x$ at $x = +1$. Show your work or you won't get *any* credit.

slope = _____

2. Let $f(x) = x^3 - 3x^2$. Use an important theorem to show that there is at least one value of c such that $f(c) = -\sqrt{2}$

3. Find the asymptotes (horizontal and vertical) of the function $y = \frac{x^2(x-2)}{2x^2-3x-2}$
Hint: The denominator factors as $(2x+1)(x-2)$.

Horizontal asymptotes: _____

Vertical asymptotes: _____

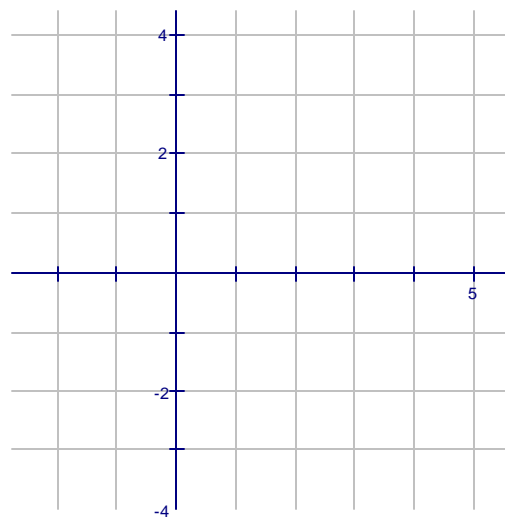
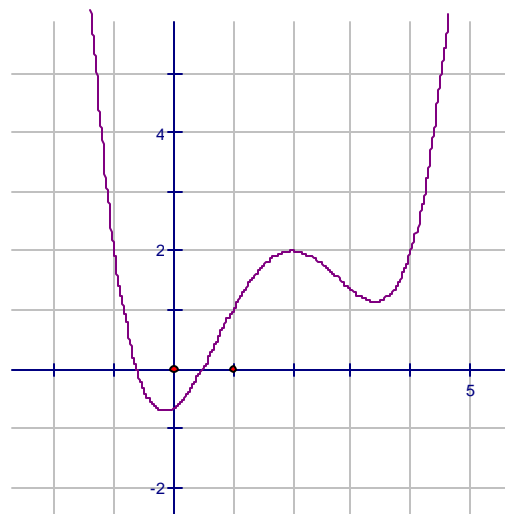
4. Find δ for $f(x) = x^2 + 3$, $L = 12$, $x_0 = 3$ and $\epsilon = 0.5$. (two decimal places will suffice.)

$\delta =$ _____

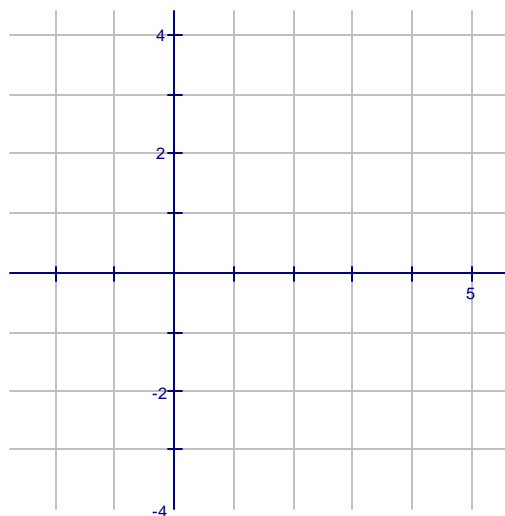
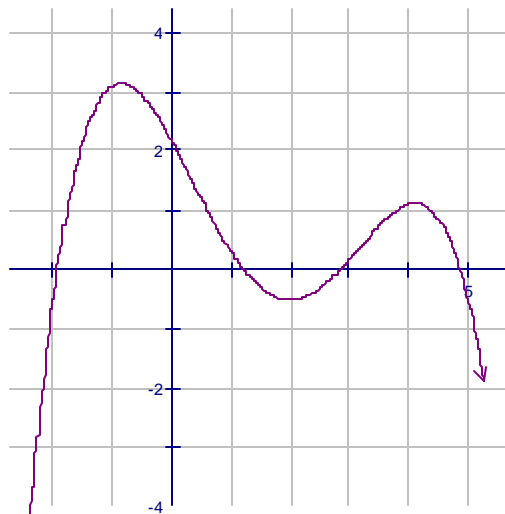
5. $f(x) = \ln \cos x$. Find $f'(x)$.

$f'(x) =$ _____

6. At the right is the graph of a function. Use the axes below to sketch the derivative of the function.



7. At the right is the graph of the derivative of a function. Use the axes below to sketch the function.



8. Use the definition of derivative to find the slope of the function $f(x) = \frac{1}{x}$ at $x = 3$.

Slope = _____

9. An object is thrown upwards in the earth's gravity with constant (downward) acceleration of -32 feet/sec^2 , from a height and with a velocity that we weren't told.

However, after seven seconds, the object was seen to hit the ground and after three seconds, it was seen to have reached its maximum height.

Hint: These tell you that $s(7) = 0$ and that $s'(3) = 0$.

Find the height from which the object was thrown _____ feet

Find the velocity with which the object was thrown _____ feet/sec

Find the speed with which the object hits the ground _____ feet/sec

Find the average speed of the body on the interval $[0,3]$ _____ feet/sec

10. Let $f(x) = \sqrt{x}$. Use the fact that $f(49) = 7$, and the things you know about the derivative of $f(x)$ to estimate $\sqrt{50}$. Compare your estimate to the actual value.

Estimate of $\sqrt{50}$ _____

Comparison: _____

11. Let $f(x) = \tan x$ on the interval $\left[0, \frac{\pi}{4}\right]$. Find the point c guaranteed by the Mean Value Theorem.

$c =$ _____

12. Let $f(x) = e^x - 2$. Use Newton's method, and an initial guess of $x = 0$ to estimate the value of x for which $f(x) = 0$. Get at least three decimal place accuracy. (Note that this value of x should be $\ln 2$.) (Show your work or you won't get *any* credit.)

Estimate: _____

13. State the First Fundamental Theorem of Calculus.

14. Use the Second Fundamental Theorem of Calculus to find the area under the curve $f(x) = x + \sin x$ and between $a = 0$ and $b = \pi$

15. Use L'Hôpital's rule to find $\lim_{x \rightarrow 0} \frac{3x - \sin x}{x}$.

limit = _____

15. Use L'Hôpital's rule to find $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1 - x/2}{x^2}$

limit = _____

16. Find the indefinite integral $\int 6xe^{x^2} dx$

17. Find the average value of the function $f(x) = e^x$ on the interval $[1,3]$

average value = _____

18. A police cruiser is approaching a right-angle intersection (along the x -axis) at 60 mph, and, at the moment, is 0.8 miles east of the intersection.

At the same time, a bad guy is fleeing the scene, going north along the y -axis, and he is 0.6 miles north of the intersection.

Just at that moment, one of the officers in the police car "shot" the fleeing bad guy with his radar gun, and found that the distance between the two cars was increasing at exactly 20 mph.

How fast was the bad guy going?

speed = _____ mph

19. We are planning to make a cylinder by rolling a rectangular piece of aluminum into a tube. We want to make the volume of the resulting cylinder as large as possible.

We are constrained by the *perimeter* of the rectangle; it has to be 144 inches.

Our problem is to find the dimensions of the rectangle that will make the cylinder have the largest possible volume.

Facts: Volume of a cylinder $V = \pi r^2 h$

r = radius of cylinder

h = height of cylinder

Circumference of a circle $C = 2\pi r$

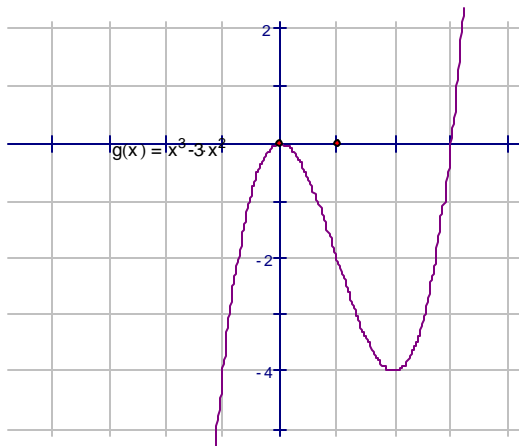
length = _____ inches

width = _____ inches

20. Simplify the expression $\sin\left(\tan^{-1}\frac{x}{\sqrt{x^2+1}}\right)$

Selected answers

2. Graph of function:
Use Intermediate Value Theorem.



9. Data:

acc	-32
vel	96
place	112

t	s(t) =	v(t) =
-1	0	128
0	112	96
1	192	64
2	240	32
3	256	0
4	240	-32
5	192	-64
6	112	-96
7	0	-128

15. (This is example 1a, page 317. Answer is 2)

15 again (This is example 2a page 317. Answer is $-1/8$)

18. This is p. 235, example 3. Answer is 70 mph.