

Assignment #16

16 3.5 p 285 # 1, 2, 6-8, 14, 16, 17, 20, 26

1. a. Row 3, column 2 = 0.15 $100(0.15) = 15$
b. Row 4, column 1 = 0.10 $40(0.10) = 4$
2. Fuels industry is most dependent on utilities since the largest element in the 4th row is in the column headed by F.
6. $1000(0.008) = 8$
7. Most dependent would be the largest entry on the main diagonal. Raw materials is the most self dependent. Likewise, Fuels is least dependent.

8. Agriculture and food (0.097)

$$14. \quad A = \begin{bmatrix} 0.3 & 0.1 \\ 0.1 & 0.2 \end{bmatrix} \quad D = \begin{bmatrix} 60 \\ 70 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \text{ where}$$

 x_1 = gross production for agriculture and x_2 = gross production for minerals.

$$X = (I - A)^{-1}D \quad I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.3 & 0.1 \\ 0.1 & 0.2 \end{bmatrix} = \begin{bmatrix} 0.7 & -0.1 \\ -0.1 & 0.8 \end{bmatrix}$$

$$(I - A)^{-1} = \frac{1}{0.56 - 0.01} \begin{bmatrix} 0.8 & 0.1 \\ 0.1 & 0.7 \end{bmatrix} \approx \begin{bmatrix} 1.45 & 0.18 \\ 0.18 & 1.27 \end{bmatrix} \quad X = (I - A)^{-1}D = \begin{bmatrix} 1.45 & 0.18 \\ 0.18 & 1.27 \end{bmatrix} \begin{bmatrix} 60 \\ 70 \end{bmatrix} = \begin{bmatrix} 99.6 \\ 99.7 \end{bmatrix}$$

Agr = 100; Min = 100

$$16. \quad A = \begin{bmatrix} 0.5 & 0.2 \\ 0.1 & 0.6 \end{bmatrix} \quad D = \begin{bmatrix} 15 \\ 33 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \text{ where}$$

 x_1 = gross production for agriculture and x_2 = gross production for steel.

$$X = (I - A)^{-1}D \quad I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.5 & 0.2 \\ 0.1 & 0.6 \end{bmatrix} = \begin{bmatrix} 0.5 & -0.2 \\ -0.1 & 0.4 \end{bmatrix}$$

$$(I - A)^{-1} = \frac{1}{0.2 - 0.02} \begin{bmatrix} 0.4 & 0.2 \\ 0.1 & 0.5 \end{bmatrix} = \begin{bmatrix} 2.2222 & 1.1111 \\ 0.5556 & 2.7778 \end{bmatrix}$$

$$X = (I - A)^{-1}D = \begin{bmatrix} 2.2222 & 1.1111 \\ 0.5556 & 2.7778 \end{bmatrix} \begin{bmatrix} 15 \\ 33 \end{bmatrix} = \begin{bmatrix} 70 \\ 100 \end{bmatrix} \text{ Agr} = 70 \text{ Steel} = 100$$

$$17. \quad D = \begin{bmatrix} 80 \\ 180 \end{bmatrix}. \quad (I - A)X = D \text{ or } \begin{bmatrix} 0.7 & -0.15 \\ -0.3 & 0.6 \end{bmatrix} \begin{bmatrix} U \\ M \end{bmatrix} = \begin{bmatrix} 80 \\ 180 \end{bmatrix}$$

$$\begin{bmatrix} U \\ M \end{bmatrix} = \begin{bmatrix} 0.7 & -0.15 \\ -0.3 & 0.6 \end{bmatrix}^{-1} \begin{bmatrix} 80 \\ 180 \end{bmatrix} = \frac{1}{0.375} \begin{bmatrix} 0.6 & 0.15 \\ 0.3 & 0.7 \end{bmatrix} \begin{bmatrix} 80 \\ 180 \end{bmatrix} = \begin{bmatrix} 200 \\ 400 \end{bmatrix} \text{ Utility} \\ \text{Manufacturing}$$

$$20. A = \begin{bmatrix} 0.20 & 0.10 \\ 0.25 & 0.45 \end{bmatrix}$$

$$D = \begin{bmatrix} 8 \\ 620 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \text{ where } x_1 = \text{gross production for agr. and } x_2 = \text{gross production for mfg.}$$

$$X = (I - A)^{-1}D \quad I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.20 & 0.10 \\ 0.25 & 0.45 \end{bmatrix} = \begin{bmatrix} 0.80 & -0.10 \\ -0.25 & 0.55 \end{bmatrix}$$

$$(I - A)^{-1} = \frac{1}{0.44 - 0.025} \begin{bmatrix} 0.55 & 0.10 \\ 0.25 & 0.80 \end{bmatrix} = \begin{bmatrix} 1.23530 & 0.24096 \\ 0.60241 & 1.92771 \end{bmatrix}$$

$$X = (I - A)^{-1}D = \begin{bmatrix} 1.23530 & 0.24096 \\ 0.60241 & 1.92771 \end{bmatrix} \begin{bmatrix} 8 \\ 620 \end{bmatrix} = \begin{bmatrix} 160 \\ 1200 \end{bmatrix} \text{ Agriculture} = 160; \text{ Manufacturing} = 1200$$

$$26. D = \begin{bmatrix} 16,500 \\ 11,400 \end{bmatrix} \quad A = \begin{bmatrix} 0 & 0.1 \\ 0.2 & 0 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad X = (I - A)^{-1}D$$

$$I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0.1 \\ 0.2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -0.1 \\ -0.2 & 1 \end{bmatrix}$$

$$(I - A)^{-1} = \frac{1}{1 - 0.02} \begin{bmatrix} 1 & 0.1 \\ 0.2 & 1 \end{bmatrix} = \begin{bmatrix} 1.02041 & 0.10204 \\ 0.20408 & 1.02041 \end{bmatrix}$$

$$X = (I - A)^{-1}D = \begin{bmatrix} 1.02041 & 0.10204 \\ 0.20408 & 1.02041 \end{bmatrix} \begin{bmatrix} 16,500 \\ 11,400 \end{bmatrix} = \begin{bmatrix} 18,000 \\ 15,000 \end{bmatrix}$$

Total costs shipping: \$18,000; Total costs printing: \$15,000.