

Numerical Analysis – Lab 16

e, ln and matrices

Goals

The goals of this lab are:

1. for you to figure out.

Preliminaries

We know from Taylor series that, if x is a real number, then

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots \text{ and that } \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \dots$$

We also know that this series for $\ln x$ converges rather slowly, and, from the last lab, we know, from the last lab, some Padé approximations that can help find $\ln x$.

We also know that $x = \ln(e^x) = e^{\ln x}$. If you have a table of logarithms, then you can find the n th root of a number by using properties of logarithms and exponentials like

$$\ln \sqrt[n]{x} = \frac{1}{n} \ln x, \text{ so that } \sqrt[n]{x} = e^{\frac{1}{n} \ln x}.$$

And we know that a non-zero number x has two square roots, a positive one and a negative one.

Matrices

Suppose we take a square matrix X instead of a real number x , and define $\ln X$ and e^X using the Taylor series above. What are the properties of the exponential and logarithm functions defined in this way?

Write it all up.

The lab is due in two weeks, April 13.