Theorem First or Example First: Newton vs Leibniz Again

Ed Sandifer
Western Connecticut State University

Newton and Leibniz disagreed on more than just who first discovered calculus. There was a basic philosophical rift over the way that science and mathematics should be done. Echoes of the dispute still ring today, and one forum for the dispute is the question indicated by the title: What should come first, the theorem or the example?
Descartes – 1596-1650

“I think. Therefore I am.”

More than an existence theorem –

A way to do science
Calculus dispute

1665/66 Newton discovered calculus
Fluents and Fluxions

1675 Leibniz discovered calculus
Differentials and Integrals

1684 Leibniz published

Never Newton published

Both claimed priority

*Philosophers at War: The quarrel between Newton and Leibniz*
A. Rupert Hall, Cambridge, 1980
<table>
<thead>
<tr>
<th>Newton</th>
<th>Leibniz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiments</td>
<td>Reason</td>
</tr>
<tr>
<td>Base proofs on truths from experience</td>
<td>Base proofs on “revealed” or logical truths</td>
</tr>
<tr>
<td>Geometry - $\dot{y}$, $\ddot{y}$, $\dddot{y}$</td>
<td>Algebra - $\frac{dy}{dx}$, $\int Xdx$</td>
</tr>
<tr>
<td>Continuity and infinite divisibility</td>
<td>Atomism and monads</td>
</tr>
<tr>
<td>God made the world for reasons we will never understand</td>
<td>God made the best of all possible worlds</td>
</tr>
</tbody>
</table>

These are generalizations and tendencies. Neither was purely “Newtonian” or “Leibnizian.”
The Battle for Euler’s Brain

Leonhard Euler

1707-1783

866 published books and articles

Three kinds of knowledge

1. “Revealed” or intuitive truth

2. Observed truth

3. Truths logically deduced from other truths

No mathematical abstraction – numbers and other objects are “real.”
Several observations about infinite series - 1737

Theorem 1: This series
\[ \frac{1}{3} + \frac{1}{7} + \frac{1}{8} + \frac{1}{15} + \frac{1}{24} + \frac{1}{26} + \frac{1}{31} + \frac{1}{35} + \text{etc.} \] equals 1.

Proof

Theorem 2: This series
\[ \frac{1}{3} + \frac{1}{7} + \frac{1}{15} + \frac{1}{31} + \frac{1}{35} + \frac{1}{63} + \text{etc.} \] equals ln 2.

Proof

Theorem 3: \( \frac{\pi}{4} \) = another series.

Proof.

A total of 19 theorems.

This at a time when Euler was “becoming more experimentalist”
Example of the use of observation in pure mathematics – 1756

Considerations about numbers contained in the form $2aa + bb$

\[
\begin{align*}
2 + bb) & \quad 3, 6, 11, 18, 27, 38, 51, 66, 83, 102, 123, \\
& \quad 146, 171, 198, 227, 258, 291, 326, 363, \\
& \quad 402, 443, 486. \\
8 + bb) & \quad 9, 17, 33, 57, 89, 129, 177, 233, 297, 369, \\
& \quad 449. \\
18 + bb) & \quad \\
32 + bb) & \quad \\
\ldots & \quad \\
392 + bb) & \quad \\
450 + bb) & \quad 451, 454, 466, 499.
\end{align*}
\]

**Observation 1**: Prime numbers on the list seem to be uniquely represented.

**Observation 2**: If a prime number is on the list, so also is its double, and only once.

\ldots

**Observation 8**: Every prime number of the form $8n+1$ and $8n+3$ appears on the list
But how do we write about mathematics?

**Theorem 1:** If $N$ is of the form $2aa + bb$, then so is its double.

**Theorem 2:** If a number $2N$ is of the form $2aa + bb$, then so also its half, $N$ is of the same form.

**Corollary 2:** Thus, if $N$ is of the form $2aa + bb$ in a unique way, then its double, $2N$, (and also $4N$, $8N$, etc.) will also be of the form in a unique way.

**Theorem 3:** If $M$ and $N$ are of the form, then so is their product $MN$.

…

**Theorem 13:** If a number $n$ is in no way the sum of a square and a triangular number, then the number $8n + 1$ certainly is not prime.
Emmanuel Kant – 1724-1804

Critique of Pure Reason – 1783

Base $a$ priori on experience

Experience tells us to believe Euclidean Geometry

Triumph of the Newtonians
What do we do in class?

When we do Theorem first
we act like rationalists

When we do Example first
we act like experimentalists

How does mathematics work?

We are experimentalists

We act like (and write like) rationalists
STORIES FROM THE HISTORY OF MATHEMATICS AS A TOOL FOR TEACHING
David Bressoud, Macalester College
Saturday, August 12, 1:00 pm – 3:00 pm

John McCleary, Vassar College, "Euler's Easy Solutions to Difficult geometric problems"

Rob Tubbs, University of Colorado, Boulder, "From e^pi to 2^(sqrt2), Motivating the Solution to Hilbert's 7th problem"

Fred Rickey, U.S. Military Academy at West Point, "Some Tested Examples for Using History in Your Classroom"

Ed Sandifer, Western Connecticut State University, "Theorem First, or Example First: Newton vs Leibniz Again."